

INFINITY

*Instant Force & Model Predictive Control for
Ocean Energy Power take-off with high Fidelity*

Deliverable 5.1

Life Model for Critical Components

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Executive Summary

As part of INFINITY WP5 “Reliability and Life Modelling”, Deliverable 5.1 deals with fatigue life modelling of the critical components of the InfinityWEC, focusing on the ball screws. The report aims to describe the developed model for assessing the fatigue life of the ball screws in the power take-off (PTO). The proposed life model is based on ISO 3408, which is reformulated to fit the purpose of tracking the accumulated fatigue damage. The observed life is modelled by a Weibull distribution, characterized by the L10-life and the Weibull exponent. Based on the Weibull assumption, the life distribution for a system of ball screws, sharing the total axial load equally, is derived. Further, a formula for the expected number of ball screw failures is developed.

The developed models and methods will be used for the further work within the INFINITY project; within WP5, to develop structural health monitoring systems (Task 5.2), and for reliability evaluations based on test rig and numerical simulation data (Task 5.3). The described model will also provide input to damage-awareness of the MPC control objective in WP2, enabling parameter optimization using the levelized cost of energy (LCoE) evaluation methods developed in WP7. Further, the report presents methods to evaluate the applicability of the fatigue life model for rig-scale testing and to assess the potential of alternative signals for condition monitoring, which serve as input to the planning of the test campaign in WP4.

Table of Contents

Executive Summary	3
Table of Contents	4
Tables	6
Figures	6
1 Introduction.....	8
1.1 Background.....	8
1.2 Ocean Harvesting Technology	8
1.3 The INFINITY Project.....	8
1.3.1 Project overview	8
1.3.2 Project objectives	9
1.4 Aim of Report.....	10
1.5 Methodology	10
1.6 Structure of Report.....	10
2 Literature Review.....	11
3 Fatigue Life Model for Ball Screws.....	12
3.1 General Model for Fatigue Life and Damage.....	12
3.1.1 Model for Life	12
3.1.2 Model for Damage Accumulation.....	12
3.2 ISO Life Model	12
3.2.1 Unidirectional Load Case	13
3.3 Adaptation of ISO Life Model to INFINITY.....	13
3.3.1 Life Model – FN-curve	14
3.3.2 Life Model – Cumulative Damage.....	14
3.3.3 Equivalent Load	15
3.3.4 An Example of Damage Accumulation	16
3.3.5 Scaling Factors for Damage and Equivalent Load	19
3.3.6 Influencing Factors on Fatigue Life.....	20
3.4 Life Distribution	21
3.4.1 Weibull Distribution.....	21
3.4.2 System Reliability	23
3.4.3 Life of System of Ball Screws	24
3.4.4 Wave Farm Reliability	26
4 Assessment of Parameters for Fatigue Model	29
4.1 NSK Ball Screw Specifications.....	29
4.2 Weibull Life Distribution.....	31
5 Load and Life Assessment.....	34
5.1 Design Load Assessment	34

5.2	Life Assessment.....	36
5.3	An Example of Load and Life Assessment for InfinityWEC G6.....	37
5.4	Fatigue Load Monitoring	41
6	Verification Strategy.....	42
6.1	Verification Approach for the Scaled PTO Testing	42
6.2	Accelerated Fatigue Testing.....	43
7	Application of Life Model to INFINITY Project	44
7.1	Life Model for Control System	44
7.2	Life Model for Structural Health Monitoring	44
7.3	Life Model for LCoE Assessment	44
8	Discussion and Conclusions	45
9	Nomenclature	46
10	References	49
Appendix A: Calculation Example.....		51
Hydrodynamic Excitation and Equation of Motion		51
Load Spectrum and Fatigue Assessment		52
Simple Sinusoidal Wave Force		52
Damage Accumulation.....		53
Degradation Classification		53
Python Code.....		54

Tables

Table 3-1: Notations for the equivalent axial load, F_m , Eq. (3.5), in the ISO standard.	13
Table 3-2: Froude model scaling.	19
Table 3-3: Quantiles and statistics for a Weibull distribution with L10-life, L , and shape parameter, $c = 1.5$	23
Table 3-4: Relative values of dynamic axial load rating, $C_{a,n}$, and life $L_{sys,n}$ for a system of n ball screws sharing the load equally, with Weibull exponent $c = 1.5$	26
Table 4-1: Model parameters used for the fatigue life model of the ball screws (provided by the product supplier, NSK). Note that the values in the table are preliminary and may differ from the final design of the full-scale and rig-scale PTO.	29
Table 4-2: Maximum allowable operational parameters for full-scale and rig-scale ball screw designs. Parameters are categorized based on whether they are ensured during the design phase (Ensured in design) or require active monitoring during operation (Monitored) to maintain safe and reliable performance.	30
Table 4-3: Uncertainty intervals for a Weibull distribution with L10-life, L , and shape parameter, $c = 1.5$	33
Table 5-1: Estimated of L10-life for the full-scale PTO design, InfinityWEC G6, for normal and tough steel with design values in Table 4-1, Weibull exponent $c = 1.5$, and one-year equivalent load $F_{eq,1} = 3000 \text{ kN}$	41
Table 9-1: Abbreviations.	46
Table 9-2: Variables.	46
Table A-1. Model parameters for estimating the loads (used for the simulation example).	53

Figures

Figure 1.1: The InfinityWEC point absorbing wave energy converter.	9
Figure 3.1: Example of fatigue assessment for 200 s of the sea state $H_s, 1.75, T_p, 6.5$. a) Total axial force acting on the PTO. b) Rotational speed of the ball screws. c) Accumulated damage, D . d) Damage intensity, D_i . For the damage calculations we assume that $C_a = 1360 \text{ kN}$	17
Figure 3.2: PDFs for Weibull distributions with different shape parameters, c , and scale parameter, $a=1$	22
Figure 3.3: PDF of a Weibull distribution with L10-life, $L = 1$, and shape parameter, $c = 1.5$. Quantiles of the distribution are shown with red lines, and the mean value with a blue dashed line.	23
Figure 3.4: Plot of relative values of dynamic axial load rating, $C_{a,n}$, for a system of n ball screws sharing the load equally, with Weibull exponent $c = 1.5$; compare with the dashed line representing a linear relation.	26
Figure 3.5: Plot of the L10-life of a wave farm, $L_{t,sys,n}(farm)$, as function of the number of WECs in the farm, n_{wec} ; the plot is in log-log-scale.	27
Figure 3.6: The expected number of failures, n_f, n, T , in a wave farm of 100 WECs, as function of the operation period T , for $n = 1, 2, 4, 6$	28
Figure 4.1: Predicted fatigue life of full-scale (left) and rig-scale (right) ball screw designs using two steel types, based on dynamic axial load ratings from NSK. Note that the curves are provided for variable mean load, but with a constant assumed mean rotational speed of 100 rpm.	30
Figure 4.2: Estimated fatigue life of the full-scale ball screw as a function of moment (left) and radial load (right). Non-shaded areas indicate safe operating conditions with minimal impact on lifetime. The results are provided by NSK.	31

Figure 4.3: PDF of a Weibull distribution with L10-life, $L = 1$, and shape parameter, $c = 1.5$. The 95% uncertainty interval is illustrated by the blue area, and quantiles (2.5%, 50%, and 97.5%) of the distribution are indicated by red lines.....33

Figure 5.1: Wave scatter diagram representing the Edvard Grieg site.35

Figure 5.2: A schematical description of the procedure for load and life assessment.35

Figure 5.3: Results from simulated 1400 s of the sea state $H_s, 1.75, T_p, 6.5$. a) Total axial force acting on PTO. b) Rotational speed of the ball screws. c) Accumulated pseudo damage, d . d) Pseudo damage intensity, di38

Figure 5.4: Plot showing the pseudo damage per sea state, neglecting occurrence, d_{ij}39

Figure 5.5: Plot showing the pseudo damage, scaled with the occurrence, $d_{ij} \cdot f_{ij}$, per sea state.39

Figure A.0.1. Results for an example simulation, for which a sinusoidal wave excitation with a force amplitude of 1000 N and frequency of 0.2 Hz is applied.52

1 Introduction

1.1 Background

This report is the result of Task 5.1 in Work Package 5 (WP5) within the INFINITY project, which focuses on improving the understanding and prediction of fatigue-related damage in critical components of power take-off (PTO) systems used in wave energy converters (WECs).

Wave energy converters are exposed to complex and variable loading conditions that significantly impact their long-term reliability and performance. Accurate fatigue modelling is therefore essential for designing robust components and ensuring operational safety. This report contributes to that goal by developing and applying a fatigue model for the ball screws, based on ISO standards, and tailored to the specific characteristics of the PTO system under investigation.

1.2 Ocean Harvesting Technology

Ocean Harvesting Technologies AB, founded in 2017, has developed the InfinityWEC: a point-absorber wave energy converter engineered for high efficiency and survivability in offshore environments. The system consists of a surface-floating buoy that moves with the waves, driving a power take-off (PTO) unit anchored near the seabed (Figure 1.1). The vertical motion of the PTO is converted into electricity using high-efficiency ball screw actuators and torque-controlled motors. A novel hydrostatic pre-tension system, which uses water pressure at depth to maintain constant tension, enables precise and instant force control. This allows the PTO to adapt in real time to varying wave conditions, maximizing energy capture while minimizing mechanical stress on the system.

1.3 The INFINITY Project

1.3.1 Project overview

The full title of the project is “Instant Force & Model Predictive Control for Ocean Energy Power take-off with high Fidelity”, with acronym INFINITY. The project is funded through the 2023 joint call of the CETPartnership, with project number CETP-2023-00478. The project is coordinated by RISE Research Institutes of Sweden AB.

The project runs for a period of three years. The formal start date may differ between the national funding agencies and their national contracts with the respective partners. The effective date of the consortium agreement is 2025-02-17. For the internal planning and follow-up of the project, we use 2025-01-01 as the start of the project.

Consortium partners (short name in parentheses; national coordinators underlined):

Sweden

- RISE Research Institutes of Sweden AB (RISE),
- Ocean Harvesting Technologies AB (OHT),
- NILU Klimat- och miljöinstitutet AB (NILU),

Italy

- VGA s.r.l. (VGA),
- Politecnico di Torino (PDT),

Ireland

- National University of Ireland Maynooth (MAU).

National Funding agencies supporting the project:

- Swedish Energy Agency (SWEA)
- Italian Ministry of Economic Development (MIMIT)
- Sustainable Energy Authority of Ireland (SAEI)

1.3.2 Project objectives

The main objective of the INFINITY project is to deliver the next-generation PTO system, along with advanced control strategies, for the InfinityWEC wave energy converter. The goal is to establish a new benchmark in wave energy by enhancing reliability, circularity, and performance. Building on validated generations of the WEC system (developed within projects SEA-50208-1, Lundin Norway, and SEA-P2022-00498), while bringing learnings and core resources from Horizon 2020 projects VALID and IMPACT, the new concept introduces key innovations such as a hydrostatic pre-tension system, enhanced ball screw actuation system and introduces power electronics from electric vehicles, including axial flux motors, SiC inverters and flywheel batteries. These advancements significantly reduce system weight while enhancing force control and survivability. The project aims to demonstrate the performance of the next-generation PTO in laboratory with realistic environmental conditions in a hardware-in-the-loop test rig setup. The next generation of the full-scale system is also developed and optimized to validate a scalable industrial supply chain, reduce the Levelized Cost of Energy (LCoE) and to reduce the environmental impact. Sustainability is promoted through circular design principles. Expected outcomes include improved reliability, deeper operational insights, reduced LCoE, and a strengthened European supply chain, marking a significant step toward commercial deployment of wave energy technologies.

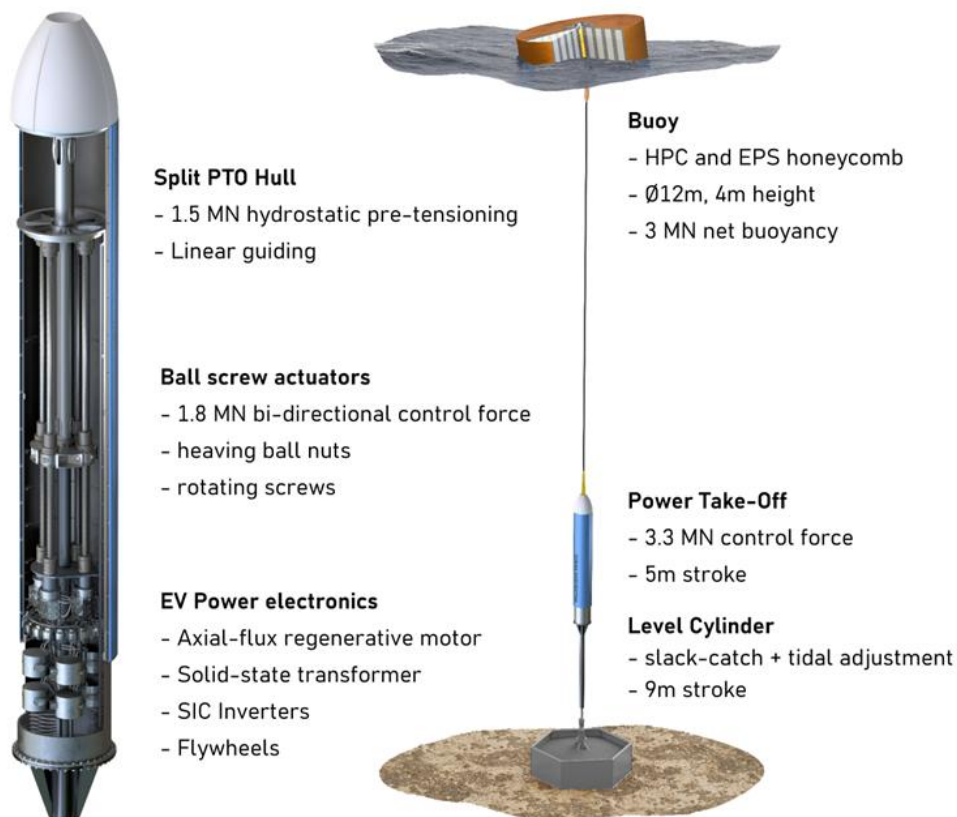


Figure 1.1: The InfinityWEC point absorbing wave energy converter.

1.4 Aim of Report

The main objective of this report, corresponding to Task 5.1, is to develop a fatigue life model capable of tracking accumulated damage of the ball screws in the PTO system. This model enables estimation of the remaining useful life (RUL) of the ball screws and is designed for enabling easy integration of damage-awareness into the model predictive control (MPC) framework developed in WP2, allowing control parameters to be optimized using the leveled cost of energy (LCoE) evaluation methods from WP7.

In addition, the report proposes a methodology for assessing the suitability of the fatigue life model for rig-scale testing and exploring alternative signals for condition monitoring, which provides input to the planning of the test campaign in WP4.

1.5 Methodology

The fatigue life model for the ball screws is developed to estimate damage accumulation over time. The primary input to the model is the number of revolutions (rotational speed) and axial force under varying amplitudes and force directions, resulting from the reciprocating motion induced by wave cycles. The modelling approach follows relevant ISO standards and will be incorporated into the MPC framework in WP2.

1.6 Structure of Report

This report is structured as follows. Chapter 2 presents a literature review that provides the theoretical background for the study. Chapter 3 describes the fatigue model developed to estimate accumulated damage over time, based on the ISO standard. In Chapter 4, the model parameters specific to the PTO system considered within the INFINITY project are summarized. Chapter 5 introduces the load and life assessment approach, followed by a discussion on verification strategy for evaluating the developed model in Chapter 6. Finally, Chapter 7 briefly presents how the fatigue model will be integrated into the related parts of the project, while Chapter 8 concludes the report with a discussion of key findings and overall conclusions.

2 Literature Review

Accurate lifetime prediction of critical components is essential for ensuring the reliability and efficiency of mechanical systems, particularly in high-demand environments such as wave energy converters (WECs). While ball screws are traditionally used in industrial automation and aerospace applications, their potential integration into WEC power take-off (PTO) systems introduces new challenges due to the harsh marine environment and highly variable loading conditions.

Lifetime prediction of ball screws has traditionally relied on classical fatigue-based models similar to those developed for rolling bearings (rolling contact fatigue theory), where the nominal life is determined using the dynamic load rating according to standards such as ISO 3408-5 (ISO 3408-5:2006), building upon (Lundberg & Palmgren, 1947). These models are widely used in design calculations and assume material fatigue as the primary failure mechanism, while computing life based on the ratio of dynamic load rating to applied load.

To account for the observed variability in ball screw lifetimes, statistical methods such as Weibull analysis have been employed to better represent the scatter in fatigue life data, inspired by methodologies originally developed for bearings (Ioannides & Harris, 1985). Building upon these, physics-based wear models have been proposed that incorporate tribological factors like lubrication, contamination, and misalignment to predict degradation rates more accurately, see e.g. (Zhao et al., 2023). Moreover, advanced models have considered the effects of thermo-mechanical interactions, recognizing that temperature-induced preload variations and vibrations can significantly influence ball screw fatigue life (Wang et al., 2020). More recently, data-driven approaches leveraging machine learning and signal processing techniques have enabled predictive maintenance by estimating the remaining useful life (RUL) of ball screws based on real-time monitoring of vibration and acoustic emissions (Sugiyama et al., 2019; Han et al., 2017).

Experimental studies support these approaches by providing labelled degradation data from long-term tests, capturing wear patterns and failure thresholds under various operating conditions (Lin et al., 2011). Zhang et al. (Zhang et al., 2018) proposed a degradation model based on fatigue wear volume, which correlates the working load and stroke count to an exponential wear function, validated through run-to-failure experiments.

Recent research has also focused on integrating lifetime prediction into control systems, particularly in Computer Numerical Control (CNC) machines and robotic applications. Health-aware control strategies utilize RUL estimations to optimize trajectories, limit acceleration or loading, and adjust motion profiles to extend component life. For example, (Denkena et al., 2023) presented a machine learning-based condition monitoring approach for ball screw drives in machine tools, using sensor data collected from a production fleet. Their method leverages automated machine learning (AutoML) and semi-supervised anomaly detection to identify degradation patterns and inform maintenance decisions, demonstrating the potential for integrating predictive models into control systems for enhanced reliability.

3 Fatigue Life Model for Ball Screws

This section describes the life model for ball screws that will be used in the INFINITY project. The model is based on the ISO standard 3408-5 which is reformulated to fit the purpose of monitoring the progress of fatigue damage. Further, additional influencing factors on life than the axial force is discussed. Finally, the Weibull distribution for life is applied to ball screws and systems thereof. However, first the general model for fatigue life and damage accumulation is presented.

3.1 General Model for Fatigue Life and Damage

3.1.1 Model for Life

The assumption (or hypothesis) is that the Wöhler curve is linear in log-log scale, which agrees with the NSK life curve and ISO 3408-5:2006. The FN-curve can thus be formulated using the Basquin equation

$$N(F) = N_0 \left(\frac{F}{F_0} \right)^{-b} \quad \text{with} \quad N_0 = 10^6 \text{ cycles} \quad (3.1)$$

where $N(F)$ is the life in number of cycles with applied positive force, F , which is typically defines as either the amplitude or the range of the cycle. The value N_0 is the defined reference number of cycles. The parameter b is the damage exponent, and (N_0, F_0) is the reference point of the Wöhler curve, where F_0 represents the fatigue strength at N_0 cycles. Here, the reference number of cycles is chosen to $N_0 = 10^6$ cycles, which is a relevant reference life for HCF (High Cycle Fatigue).

3.1.2 Model for Damage Accumulation

Assume that the load can be represented by a sequence of cycles, which gives a sequence of positive force values, F_1, F_2, \dots, F_n , where F_i is the force value of the i :th cycle. The cycles are typically assessed using the rainflow cycle method. The damage due to the load can be calculated using the Palmgren-Miner damage accumulation rule together with the Wöhler curve for force

$$D = \sum_{i=1}^n \frac{1}{N(F_i)} = \frac{1}{N_0 \cdot F_0^b} \cdot \sum_{i=1}^n F_i^b = \frac{1}{N_0 \cdot F_0^b} \cdot d \quad (3.2)$$

where

$$d = \sum_{i=1}^n F_i^b \quad (3.3)$$

is the so-called pseudo damage. The damage represents the fraction of consumed life. The pseudo damage represents the accumulated fatigue load.

General information on load analysis, e.g. rainflow cycles and damage accumulation can be found in e.g. (Johannesson & Speckert, 2014).

3.2 ISO Life Model

The International Organization for Standardization (ISO) has a standard for ball screws called ISO 3408. This standard is divided into multiple parts, each addressing specific aspects of ball screws. Part 5 (ISO 3408-5:2006) is titled "Static and dynamic axial load ratings and operational lifetime". This part of ISO 3408 specifies the calculation schemes for static and

dynamic load ratings, and operational life, in order to obtain comparable values for the design and use of ball screws.

3.2.1 Unidirectional Load Case

For ball screw with backlash between ball nut and screw shaft and unidirectional applied external axial load, ISO 3408-5:2006 states the formula for life, L , in number of revolutions

$$L = \left(\frac{C_a}{F_m} \right)^3 \cdot 10^6 \quad (3.4)$$

where C_a is the basic dynamic axial load rating and F_m is the equivalent axial load.

For the case of variable axial load and variable rotational speed, the standard provides formulas for equivalent (mean) values of the axial force F_m and rotational speed ω_m ¹. In particular, for the case of variable axial load and variable rotational speed, the equivalent values of the axial load, F_m , rotational speed, ω_m , should be calculated as

$$F_m = \sqrt[3]{\sum_{j=1}^n F_j^3 \cdot \frac{\omega_j}{\omega_m} \cdot \frac{q_j}{100}} \quad (3.5)$$

and

$$\omega_m = \sum_{j=1}^n \omega_j \frac{q_j}{100} \quad (3.6)$$

where q_j is the subpart in percent for which the rotational speed corresponds to ω_j , see Table 3-1 for the other notations. Note that the sum in Eq. (3.5) corresponds to damage accumulation of the variable amplitude load.

Table 3-1: Notations for the equivalent axial load, F_m , Eq. (3.5), in the ISO standard.

Notation	Description	Unit
n	Number of load cases	-
F_j	Axial load for load case j	kN
ω_j	Rotational speed for load case j	rpm
ω_m	Rotational speed mean value	rpm
q_j	Time proportion for load case j	%

3.3 Adaptation of ISO Life Model to INFINITY

Now a life model for ball screws will be formulated based on the general life model in Section 3.1 and the ISO model, ISO 3408-5:2006, which better suits the needs of the INFINITY project. The incremental damage for each time period can be calculated, and the accumulated damage can easily be updated.

¹ The SI unit for rotational speed (or angular velocity) is radians per second (rad/s). However, in mechanical engineering and practical applications (like motors, ball screws, etc.), rpm and rps (i.e. revolutions per minute/second) are often more intuitive and commonly used.

3.3.1 Life Model – FN-curve

Define the FN-curve according to ISO 3408-5:2006, i.e. the Basquin equation

$$L(F) = N_0 \left(\frac{F}{C_a} \right)^{-b} \quad \text{with} \quad N_0 = 10^6 \text{ cycles} \quad (3.7)$$

where $L(F)$ is the life in number of cycles with applied axial force F , and N_0 is the defined reference number of cycles. Note that for bidirectional axial loading (as this case), the absolute value of the force is used. The parameter b is the damage exponent, and (N_0, C_a) is the reference point of the Basquin curve, where C_a is the basic dynamic axial load rating, which represents the fatigue strength at N_0 cycles. Following ISO 3408-5:2006, the exponent is set to $b = 3$, and the reference number of cycles is chosen to $N_0 = 10^6$ cycles.

3.3.2 Life Model – Cumulative Damage

Assume that the load during period $[0, T]$ can be represented by a sequence of force values, F_1, F_2, \dots, F_n , and a sequence of rotational speeds $\omega_1, \omega_2, \dots, \omega_n$, where F_i and ω_i are the absolute values of axial force and the rotational speed, respectively, during the i :th time interval of duration Δt_i . The rotational speed is here defined as the number of revolutions per time unit. It is worth noting that the quantities ω_i and Δt_i should be defined in the same unit of time. Further, note that the axial speed of the shaft can be calculated from the rotational speed as

$$v_i^{(axial)} = P_h \omega_i \quad (3.8)$$

where P_h is the lead of one revolution.

The damage due to the load can be calculated using Palmgren-Miner damage accumulation rule together with the FN-curve, giving

$$D = \sum_{i=1}^n \frac{n_i}{N(F_i)} = \frac{1}{N_0 \cdot C_a^b} \cdot \sum_{i=1}^n n_i \cdot F_i^b \quad (3.9)$$

where n_i is the number of cycles during the i :th time interval of duration Δt_i . Now, n_i can be calculated as

$$n_i = \omega_i \cdot \Delta t_i \quad (3.10)$$

Combing the above equations give

$$D = \frac{1}{N_0 \cdot C_a^b} \cdot \sum_{i=1}^n \omega_i \cdot \Delta t_i \cdot F_i^b \quad (3.11)$$

The estimated L10-life in years can now be estimated as

$$L_t = \frac{T}{D} \quad (3.12)$$

where the L_t is in years (or more precisely, the same unit as T).

Further, the pseudo damage is defined as

$$d = \sum_{i=1}^n \omega_i \cdot \Delta t_i \cdot F_i^b = \sum_{i=1}^n d_i \quad (3.13)$$

where d_i represents the increase in pseudo damage during the i :th time interval, Δt_i . Note that the pseudo damage only depends on the applied load and not on the load capacity parameter, C_a . Thus, the accumulated fatigue load can be represented by the pseudo damage that can be monitored independently of the specific dynamic axial load rating of the ball screw.

For structural health monitoring and for health-aware control strategies it can be of interest to study the damage increase per time unit, denoted by the damage intensity. The damage intensity, \tilde{D}_i , for the i :th sampling interval, can be evaluated as

$$\tilde{D}_i = \frac{\omega_i \cdot F_i^b}{N_0 \cdot C_a^b} = \frac{\tilde{d}_i}{N_0 \cdot C_a^b} \quad (3.14)$$

where

$$\tilde{d}_i = \omega_i \cdot F_i^b \quad (3.15)$$

is the pseudo damage intensity calculated at the i :th sampling interval.

3.3.3 Equivalent Load

The pseudo damage can be difficult to interpret due to its unit of kN^3 . Therefore, a damage equivalent load will here be defined in terms of an equivalent force and its number of cycles. More precisely, the equivalent load is a constant amplitude load, which is defined by the equivalent force, F_{eq} , together with an equivalent number of cycles, N_{eq} . The equivalent number of cycles is here chosen to a fixed number of cycles, $N_{eq} = 10^6$. The equivalent force, F_{eq} , thus represents the magnitude of the equivalent load, and N_{eq} , the number of cycles.

The equivalent load shall be damage equivalent to the expected fatigue load during the design life, T_{design} . Denote by D_T , the damage for a load of duration T . The damage extrapolated to the design life thus becomes

$$D_{design} = \frac{T_{design}}{T} \cdot D_T = \frac{T_{design}}{T} \cdot \frac{d_T}{N_0 \cdot C_a^b} \quad (3.16)$$

where d_T is the pseudo damage for duration T . Further, the damage due to the equivalent load is evaluated as

$$D_{eq} = \frac{F_{eq}^3 \cdot N_{eq}}{N_0 \cdot C_a^b} \quad (3.17)$$

Solving the damage equivalence equation, $D_{eq} = D_{design}$, gives the equivalent force

$$F_{eq} = \left(\frac{T_{design} \cdot d_T}{T \cdot N_{eq}} \right)^{1/3} \quad (3.18)$$

Note that the equivalent load does not depend on the load rating, C_a , but only depends on the accumulated load. Further, since the equivalent number of cycles has been chosen to the same value as the reference number of cycles in the FN-curve, i.e., $N_{eq} = N_0 = 10^6$, the

equivalent force, F_{eq} , can be directly compared to the load rating, C_a . The separation of the load variable, F_{eq} , from the strength (or load rating) variable, C_a , is an important feature of the equivalent load formulation.

For life assessment, it can be useful to also define an equivalent load representing one year of usage. The one-year equivalent load becomes

$$F_{eq,1} = \left(\frac{T_1 \cdot d_T}{T \cdot N_{eq}} \right)^{1/3} \quad (3.19)$$

with $T_1 = 1$ year.

Further, for structural health monitoring, it can be useful to track the evolution of accumulated pseudo damage. The pseudo damage, d_T , accumulated during period T can then be transformed into a corresponding equivalent load, $F_{eq,T}$, according to

$$F_{eq,T} = \left(\frac{d_T}{N_{eq}} \right)^{1/3} \quad (3.20)$$

Thus, it is possible to follow the evolution of the equivalent load, $F_{eq,T}$, as function of operation time, T , and compare $F_{eq,T}$ to the load rating, C_a , in order to evaluate the remaining safety margin.

Note that the L10-life can be estimated from any of the formulations of the equivalent load, namely

$$L_t = \left(\frac{C_a}{F_{eq}} \right)^3 \cdot T_{design} = \left(\frac{C_a}{F_{eq,1}} \right)^3 \cdot T_1 = \left(\frac{C_a}{F_{eq,T}} \right)^3 \cdot T \quad (3.21)$$

where the life is in years.

3.3.3.1 Equivalent Load by ISO

Recall that for a specified design load spectrum, ISO 3408-5:2006 defines an equivalent axial load, F_m , together with a mean rotational speed, ω_m , which corresponds to an equivalent load with

$$F_{eq,ISO} = F_m \quad \text{and} \quad N_{eq,ISO} = T_{design} \cdot \omega_m \quad (3.22)$$

Thus, for the equivalent load defined by the ISO standard, both the force and the number of cycles, $(F_{eq,ISO}, N_{eq,ISO})$, depend on the design load specification.

However, the ISO equivalent load description can easily be translated into an equivalent load with fixed number of cycles, N_{eq} , according to

$$F_{eq} = F_m \cdot \left(\frac{T_{design} \cdot \omega_m}{N_{eq}} \right)^{1/3} \quad (3.23)$$

3.3.4 An Example of Damage Accumulation

To demonstrate the life model and damage accumulation, a short interval of a sea state (quantifying how wave-induced motion translates into mechanical forces acting on the PTO) is used. The input for the sea state is received from simulations made by the detailed model

developed in WP2. Figure 3.1a-b shows the axial force acting on the PTO and the rotational speed of the individual ball screws over 200 seconds of the load spectrum for the sea state $(H_{s,1.75}, T_{p,6.5})$.

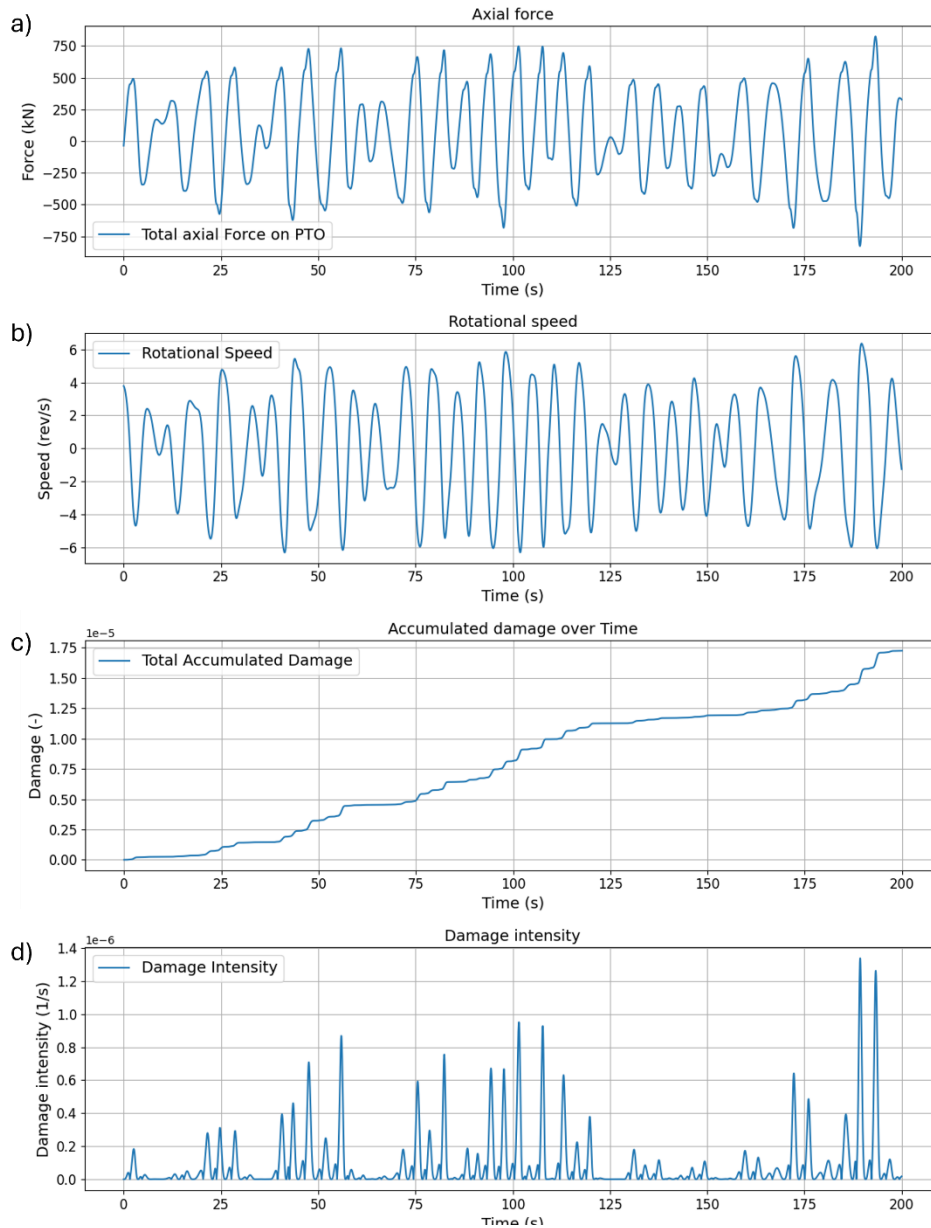


Figure 3.1: Example of fatigue assessment for 200 s of the sea state $(H_{s,1.75}, T_{p,6.5})$. a) Total axial force acting on the PTO. b) Rotational speed of the ball screws. c) Accumulated damage, D . d) Damage intensity, \tilde{D}_i . For the damage calculations we assume that $C_a = 1360$ kN.

Further, Figure 3.1c-d shows the accumulated damage, D , as function of time (assuming $C_a = 1360$ kN) and the corresponding damage intensity, \tilde{D}_i , at each sampling interval Δt

$$\tilde{D}_i = \frac{\omega_i \cdot F_i^3}{N_0 \cdot C_a^3} \quad (3.24)$$

Note that the damage intensity is proportional to the cube of the force and directly proportional to the rotational speed. As expected, this can be seen from the results in Figure 3.1d, i.e. the

damage contribution is very large for high forces. Furthermore, note that high forces often occur at high rotational speeds, which further increases the damage increment.

The accumulated damage over the period $T = 200$ s is $D = 1.75 \cdot 10^{-5}$. Thus, assuming the load represents the operational loads, the estimated L10-life is

$$L_t = \frac{T}{D} = \frac{200 \text{ s}}{1.75 \cdot 10^{-5}} = 11 \cdot 10^6 \text{ s} = 0.37 \text{ years} \quad (3.25)$$

This value should be compared to the design life of the ball screws, which is the planned maintenance intervals, here assumed to be $T_{design} = 5$ years. Obviously, in this example, the estimated life is too short compared to the required life, which can be resolved either by increasing the strength of the ball screw or by reducing the loads on the ball screw, see also further comments below.

Another way to estimate the L10-life is by deriving the pseudo damage and the corresponding equivalent load as follows. For the given loading cycle, the pseudo damage for the corresponding 200 s is calculated as

$$d_T = \sum_{i=1}^n \omega_i \cdot \Delta t_i \cdot F_i^b = 4.3 \cdot 10^{19} \text{ N}^3 \quad (3.26)$$

Based on the pseudo damage, the equivalent load can now be estimated as

$$F_{eq} = \left(\frac{T_{design} \cdot d_T}{T \cdot N_{eq}} \right)^{1/3} = \left(\frac{5 \cdot 365 \cdot 24 \cdot 3600 \text{ s} \cdot 4.3 \cdot 10^{19} \text{ N}^3}{200 \text{ s} \cdot 10^6} \right)^{1/3} = 3236 \text{ kN} \quad (3.27)$$

The life in years (for the given value of $C_a = 1360$ kN) can now be estimated as follows

$$L_t = \left(\frac{C_a}{F_{eq}} \right)^3 \cdot T_{design} = \left(\frac{1360 \cdot 10^3}{3236 \cdot 10^3} \right)^3 \cdot 5 \text{ years} = 0.37 \text{ years} \quad (3.28)$$

The life estimate can equally well be performed based on the one-year pseudo damage, according to

$$F_{eq,1} = \left(\frac{T_1 \cdot d_T}{T \cdot N_{eq}} \right)^{1/3} = \left(\frac{5 \cdot 365 \cdot 24 \cdot 3600 \text{ s} \cdot 4.3 \cdot 10^{19} \text{ N}^3}{200 \text{ s} \cdot 10^6} \right)^{1/3} = 1893 \text{ kN} \quad (3.29)$$

giving the L10-life estimate

$$L_t = \left(\frac{C_a}{F_{eq,1}} \right)^3 \cdot T_1 = \left(\frac{1360 \cdot 10^3}{1893 \cdot 10^3} \right)^3 \cdot 1 \text{ years} = 0.37 \text{ years} \quad (3.30)$$

The life in years (for the given value of $C_a = 1360$ kN) can now be estimated as follows

Note that both Eq. (3.28) and Eq. (3.30) can be reformulated into the equation

$$L_t = \left(\frac{C_a}{F_{eq,T}} \right)^3 \cdot T \quad (3.31)$$

with $F_{eq,T}$ defined in Eq. (3.20).

From these results, it can be seen that the different methods yield the same estimated life, since they are merely reformulations of the same calculation. It should be noted, however, that

this example only considers a short period of a single sea state out of all the possible sea states to which the PTO is expected to be exposed during operation. To obtain a more representative estimate of the lifetime, the full sea state distribution for the site of operation, as well as the operating conditions of the WEC, should be taken into account. A more detailed example taking all sea states into account is provided in Section 5.3.

3.3.5 Scaling Factors for Damage and Equivalent Load

During WEC development, it is common to work with scaled prototypes. Thus, there is a need for scaling rules, where the Froude model scaling is used, see Table 3-2. The scaling of damage, pseudo damage and equivalent load will here be derived.

Table 3-2: Froude model scaling.

Property	Scaling
Length	λ
Speed	$\lambda^{0.5}$
Force	λ^3
Power	$\lambda^{3.5}$

Now consider a ball screw with lead P_h , and a scaled ball screw with scale factor λ and lead $P_{h,\lambda}$. Froude scaling is applied on force, $F_{\lambda,i} = \lambda^3 F_i$, and axial speed $v_{\lambda,i}^{(axial)} = \lambda^{0.5} v_i^{(axial)}$. When calculating the damage of the scaled load, according to Eqs. (3.9)-(3.11), both the number of cycles and the force is affected by the scaling. The number of cycles for the non-scaled and scaled load during the sampling interval i is expressed as

$$n_i = \omega_i \cdot \Delta t_i \quad \text{with} \quad v_i^{(axial)} = P_h \omega_i \quad (3.32)$$

$$n_{\lambda,i} = \omega_{\lambda,i} \cdot \Delta t_i \quad \text{with} \quad v_{\lambda,i}^{(axial)} = P_{h,\lambda} \omega_{\lambda,i} \quad (3.33)$$

Utilizing the scaling of speed, the number of cycles for scaled load becomes

$$n_{\lambda,i} = \lambda^{0.5} \cdot \frac{P_h}{P_{h,\lambda}} \cdot n_i \quad (3.34)$$

The pseudo damage, $d_{\lambda,i}$, of the scaled load can now be expressed in terms of the pseudo damage, d_i , of the non-scaled load as

$$d_{\lambda,i} = n_{\lambda,i} \cdot F_{\lambda,i}^3 = \lambda_n \cdot \lambda_F^3 \cdot d_i \quad (3.35)$$

with

$$\lambda_n = \lambda^{0.5} \cdot \frac{P_h}{P_{h,\lambda}} \quad \text{and} \quad \lambda_F = \lambda^3 \quad (3.36)$$

where the first scaling factor, λ_n , is the scaling of the number of cycles, depending on the speed scaling and the difference in lead, while the second scale factor, λ_F , is the force scaling.

Thus, the damage, D , and the pseudo damage, d , is scaled according to

$$\lambda_d = \frac{D_\lambda}{D} = \frac{d_\lambda}{d} = \lambda_n \cdot \lambda_F^3 = \lambda^{9.5} \cdot \frac{P_h}{P_{h,\lambda}} \quad (3.37)$$

and the equivalent load is scaled as

$$\lambda_{F_{eq}} = \frac{F_{eq,\lambda}}{F_{eq}} = \lambda_n^{1/3} \cdot \lambda_F = \left(\lambda^{0.5} \cdot \frac{P_h}{P_{h,\lambda}} \right)^{1/3} \cdot \lambda^3 \quad (3.38)$$

3.3.6 Influencing Factors on Fatigue Life

The fatigue life of the ball screw depends on several factors. To account for the effects of, e.g., radial forces, moments, or (dynamic) operation with shock loadings, the load factor f_w can be introduced as follows

$$L(F) = N_0 \left(\frac{F \cdot f_w}{C_a} \right)^{-b} \quad (3.39)$$

The primary load in this case is the axial load (F); however, provided that relevant data is available, an extended load factor may be expressed as the multiplicative product of the individual components

$$f_w = f_w^r \cdot f_w^m \cdot f_w^{sh} \cdot f_w^{th} \quad (3.40)$$

Here, f_w^r , f_w^m , f_w^{sh} , and f_w^{th} correspond to correction factors related to radial forces, moment, shock loads, and temperature (or lubrication factor), respectively. To ensure safe operation in a highly dynamic environment, it is recommended to use a value of $f_w > 1$, e.g., within the range 1.2-3.5 according to (THK Co., Ltd.).

Proper lubrication of the ball screw is critical for reliable operation. Due to the limited accessibility of the PTO during service, lubricant replenishment is not feasible (or practical), making it essential to maintain a dry and particle-free environment around the screws. Additionally, elevated temperatures reduce the strength of the lubricant's oil film, increasing the risk of inadequate lubrication and potential failure. To mitigate this risk, it is recommended to keep the operating temperature below 70 °C (θ_{max}), measured at the nut diameter (NSK Ltd., 2023). Continuous temperature monitoring during operation is advised to ensure this limit is not exceeded. Moreover, to ensure safe operation of the ball screw, the operating rotational speed, ω_i , should remain below the critical speed limit ω_{max} . This limit is determined by the screw's design and is provided by the supplier. Continuous monitoring of ω_i is also recommended to prevent overspeed during operation.

It is also possible to adjust the reliability of the life estimate by applying the so-called reliability factor f_{ar} as follows

$$L_{ar} = L \cdot f_{ar} \quad (3.41)$$

For a reliability level of 90%, the factor f_{ar} is set to 1. For other reliability levels, we refer to the ISO-standard (ISO 3408-5:2006) and Section 4.2.

Finally, it is worth noting that the ISO standard (ISO 3408-5:2006) introduces additional correction factors related to the screw ball design, including aspects such as geometry and material processing. In particular, a modified dynamic axial load rating, C_{am} , is suggested as follows

$$C_{am} = C_a \cdot f_h \cdot f_{ac} \cdot f_m \quad (3.42)$$

where f_h , f_{ac} , and f_m represent corrections for surface hardness, manufacturing accuracy and influence of the material melting process, respectively. In this project, these considerations are assumed to be incorporated into the dynamic axial load ratings, C_a , provided by the supplier.

3.4 Life Distribution

3.4.1 Weibull Distribution

In order to model the variability in operational life, a statistical distribution can be used. For ball bearings and ball screws a Weibull distribution with two parameters can be used, with cumulative distribution function (CDF)

$$F_X(x) = 1 - \exp\left(-\left(\frac{x}{a}\right)^c\right) \quad (3.43)$$

where c is the shape parameter, called the Weibull exponent, and a is the scale parameter, called the characteristic life.

The mean value and standard deviation of a Weibull distribution is

$$m_W = a \cdot \Gamma\left(1 + \frac{1}{c}\right), \quad s_W = a \cdot \sqrt{\Gamma\left(1 + \frac{2}{c}\right) - \Gamma^2\left(1 + \frac{1}{c}\right)} \quad (3.44)$$

and with a relative standard deviation (coefficient of variation) of

$$\frac{s_W}{m_W} = \sqrt{\frac{\Gamma\left(1 + \frac{2}{c}\right)}{\Gamma^2\left(1 + \frac{1}{c}\right)} - 1} \quad (3.45)$$

that is independent of the scale parameter, a , and only depends on the Weibull exponent c . Examples of probability distribution functions (PDFs) for Weibull distributions with different shape parameters are shown in Figure 3.2. Note that $c = 1$ corresponds to the exponential distribution, which corresponds to a constant failure rate. The Weibull distribution has an increasing failure rate if $c > 1$, which is the typical case for applications like fatigue life.

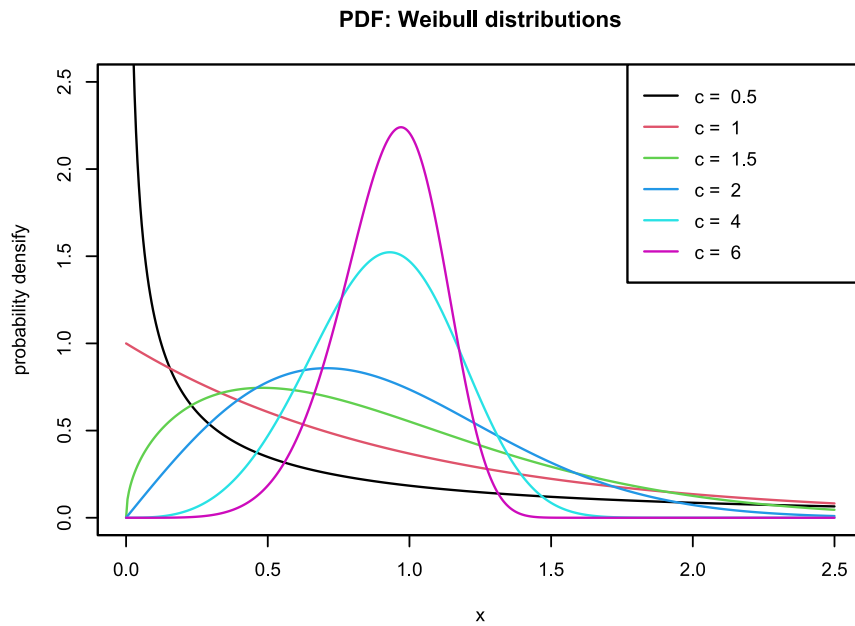


Figure 3.2: PDFs for Weibull distributions with different shape parameters, c , and scale parameter, $a=1$.

The ISO standard 3408-5 gives the life, L , corresponding to 90% reliability. However, the Weibull distribution is parametrized by the characteristic life, a , which represents 36.8% reliability. Parameter a can be calculated from L by solving the equation $0.9 = 1 - F_X(L)$, giving

$$a = \frac{L}{(-\ln 0.9)^{1/c}} \quad (3.46)$$

For a Weibull exponent of $c = 1.5$, it results in $a = 4.48 \cdot L$. Further, any quantile, x_p , of the Weibull distribution can be obtained by solving the equation $F_X(x_p) = p$, giving

$$x_p = a(-\ln(1 - p))^{1/c} = L \left(\frac{\ln(1 - p)}{\ln 0.9} \right)^{1/c} \quad (3.47)$$

The Weibull life distribution is illustrated in Figure 3.3, with L10-life, $L = 1$, and Weibull shape parameter, $c = 1.5$, representative for ball screws, see Section 4.2. Quantiles of the distribution corresponding to 10%, 50%, and 90% failure probabilities are indicated with red vertical lines, and also the characteristic life, a , is marked, corresponding to 63.2% failure probability. The mean value is indicated by a blue dashed line. The values of quantiles and statistics for a Weibull distribution with shape parameter, $c = 1.5$, and L10-life, L , is tabulated in Table 3-3. Note that all values (except the L10-life) depend on the Weibull shape parameter. Further, the quantiles, the mean and the standard deviation scales with the L10-life, while the relative standard deviation only depends on the Weibull shape parameter.

PDF: Weibull distribution with L10=1, and c=1.5

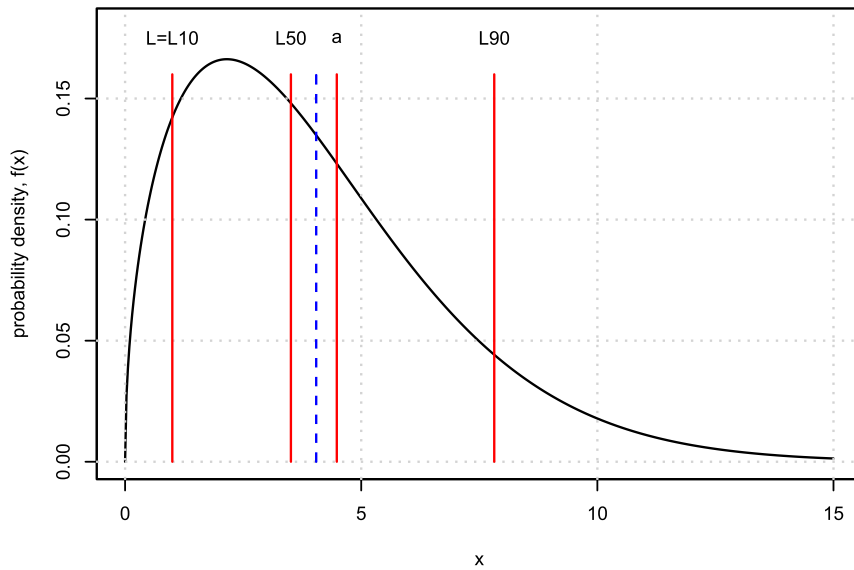


Figure 3.3: PDF of a Weibull distribution with L10-life, $L = 1$, and shape parameter, $c = 1.5$. Quantiles of the distribution are shown with red lines, and the mean value with a blue dashed line.

Table 3-3: Quantiles and statistics for a Weibull distribution with L10-life, L , and shape parameter, $c = 1.5$.

Parameter	Quantiles				Statistics		
	10%-quantile	50%-quantile	90%-quantile	Characteristic life	Mean value	Standard deviation	Relative standard deviation
Notation	L10	L50	L90	a	m_W	s_W	$\frac{s_W}{m_W}$
Value	L	$3.51 \cdot L$	$7.82 \cdot L$	$4.48 \cdot L$	$4.05 \cdot L$	$2.75 \cdot L$	0.679

3.4.2 System Reliability

In this section, general system reliability based on the Weibull distribution will be discussed. In the following sections system reliability will be discussed in two different settings relevant to the InfinityWEC, namely:

1. Load capacity for a system of ball screws sharing the load, Sec. 3.4.3.
2. Reliability of a wave farm, Sec. 3.4.4.

Now assume that all components need to work for the system to work, then the life of the system is determined by the shortest life of the individual components. In terms of system reliability this represents a series system, where the shortest life determines the system life. This is also referred to as the weakest link principle. In case all components have the same life distribution, it is easy to formulate the distribution of the weakest link.

A system reliability framework for mooring and foundations was developed in (DTOcean, 2015) together with a software tool. System reliability evaluation for wave energy applications was investigated in (WaveBoost D6.3, 2019). General statistical theory for system reliability can be found in, e.g., (Rausand & Høyland, 2004).

Now consider a series system where the minimum life determines the system life. The life of individual components is modelled by random variables, X_1, X_2, \dots, X_n each with cumulative distribution function $F_X(x) = P(X_i \leq x)$, and reliability function $R_X(x) = 1 - F_X(x)$. The minimum of n independent such variables has the reliability function

$$P(\min X_i > x) = P(\text{all } X_i > x) = \prod_{i=1}^n P(X_i > x) = R(x)^n = (1 - F_X(x))^n \quad (3.48)$$

which represents the distribution for the weakest link.

For a series system of components with independent Weibull distributed life, all having the same parameters, the result is a distribution for minimum life, with reliability function

$$R_{X_{min}}(x) = \exp\left(-\sum_{i=1}^n \left(\frac{x}{a}\right)^c\right) = \exp\left(-n \left(\frac{x}{a}\right)^c\right) = \exp\left(-\left(\frac{x}{a_n}\right)^c\right) \quad (3.49)$$

resulting in a Weibull distribution with same exponent, c , but with shorter characteristic life, a_n , given by

$$a_n = a \cdot n^{-1/c} \quad (3.50)$$

This is the main result that will be used below for modelling the reliability of a system of ball screws and for evaluating the reliability of a wave farm.

It is also possible to derive the minimum distribution for Weibull distributions with individual parameters, and for completeness the corresponding results are presented below. For a series system of independent components with same Weibull exponent but individual a_i -parameters, the reliability function for minimum becomes

$$R_{X_{min}}(x) = \exp\left(-\sum_{i=1}^n \left(\frac{x}{a_i}\right)^c\right) \quad (3.51)$$

resulting in a Weibull distribution with

$$a_n = \left(\sum_{i=1}^n \frac{1}{a_i^c}\right)^{-1/c} \quad (3.52)$$

The general expression for the distribution of minimum for a series system of independent Weibull components is given by the reliability function

$$R_{X_{min}}(x) = \exp\left(-\sum_{i=1}^n \left(\frac{x}{a_i}\right)^{c_i}\right) \quad (3.53)$$

where (a_i, c_i) are the Weibull parameters for component i . Now the minimum is no longer a Weibull distribution, but is called a poly-Weibull distribution, (Berger & Sun, 1993).

3.4.3 Life of System of Ball Screws

In the InfinityWEC the total axial load is shared between multiple ball screws in order to be able to cope with the total axial load. The current design of the INFINITY PTO consists of four

ball screws sharing the load equally. There is a discussion in the design process on the ideal number of ball screws in each WEC. This section presents results on the life and load capacity for a system of ball screws sharing the load equally. Note that life here denotes the time until first failure of the ball screws in the WEC, and that the ball screws are wear parts that are planned for replacement, either at failure or at regular maintenance intervals during the lifetime of the WEC.

The life of the ball screw system of a WEC is determined by the shortest life of the ball screws in the WEC, thus the weakest link principle applies. In case the load is shared equally between the components, and they all have the same load rating, C_a , the distribution for the life can be formulated based on the weakest link principle.

First consider a WEC with only one ball screw that is exposed to total load F_1 , then the L10-life becomes

$$L_1 = 10^6 \cdot \left(\frac{C_a}{F_1}\right)^3 \quad (3.54)$$

Now consider a system of n ball screws, where the total load, F_1 , is shared equally between the ball screws. The load on each ball screw becomes $F_n = F_1/n$. Therefore, the L10-life of each of the ball screws is

$$L_n = 10^6 \cdot \left(\frac{C_a}{F_n}\right)^3 = 10^6 \cdot \left(\frac{nC_a}{F_1}\right)^3 = L_1 \cdot n^3 \quad (3.55)$$

The L10-life for a system of n ball screws sharing the load is now obtained using the weakest link principle

$$L_{sys,n} = L_n \cdot n^{-1/c} = L_1 \cdot n^3 \cdot n^{-1/c} = L_1 \cdot n^{3-1/c} \quad (3.56)$$

Further, note that the life of a ball screw is proportional to the cube of the dynamic axial load rating, C_a . Thus, the dynamic axial load rating, $C_{a,n}$, for a system of n ball screws with shared load can be calculated as

$$C_{a,n} = \left((nC_a)^3 \cdot n^{-1/c}\right)^{1/3} = C_a \cdot n^{1-\frac{1}{3c}} \quad (3.57)$$

where nC_a is due to the load sharing, and $n^{-1/c}$ is due to weakest link.

The above relations will be illustrated by an example using a typical value of the Weibull exponent for ball screws of $c = 1.5$. The dynamic axial load rating, $C_{a,n}$, and the L10-life, $L_{sys,n}$, for a system of n ball screws is calculated to

$$C_{a,n} = C_a \cdot n^{1-\frac{1}{3c}} = C_a \cdot n^{7/9} \quad \text{and} \quad L_{sys,n} = L_1 \cdot n^{3-1/c} = L_1 \cdot n^{7/3} \quad (3.58)$$

and the relations are shown in Table 3-4 and Figure 3.4. Thus, the dynamic axial load rating for a system of ball screws does not scale linearly with the number of ball screws (indicated by the dashed line), but increases at a slightly slower rate, namely proportional to $n^{7/9}$. As can be seen in Table 3-4, the dynamic axial load rating for a system of 4 ball screws is not $4C_a$ but approximately $3C_a$, which is due to the weakest link principle; the load rating for a system of 6 ball screws is about $4C_a$.

Table 3-4: Relative values of dynamic axial load rating, $C_{a,n}$, and life $L_{sys,n}$ for a system of n ball screws sharing the load equally, with Weibull exponent $c = 1.5$.

Number of ball screws, n	1	2	3	4	5	6
Relative rating, $C_{a,n}/C_a$	1	1.71	2.35	2.94	3.50	4.03
Relative life, $L_{sys,n}/L_1$	1	5.04	12.98	25.40	42.75	65.42

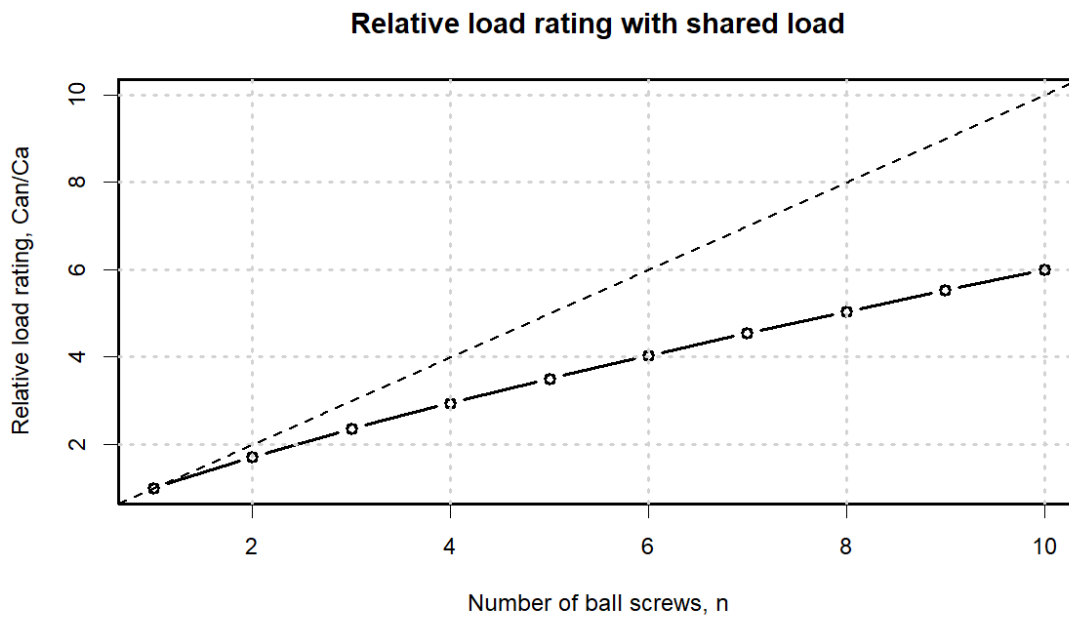


Figure 3.4: Plot of relative values of dynamic axial load rating, $C_{a,n}$, for a system of n ball screws sharing the load equally, with Weibull exponent $c = 1.5$; compare with the dashed line representing a linear relation.

3.4.4 Wave Farm Reliability

In this section the reliability of ball screws in a wave farm will be discussed, and two topics will be covered:

1. time until first failure, and
2. expected number of failures.

Assume that all WECs in the wave farm experience the same wave climate and thus have the same average fatigue load. The life of a ball screw system for a WEC with n ball screws is modelled by a Weibull distribution with parameters a_n and c . Denote by $L_{t,sys,n}$ the L10-life in years for a system of n ball screws sharing the load equally, see Eq. (3.56).

3.4.4.1 Time until First Failure

The reliability of a wave farm can be modelled as a series system, and the time until the first failure occurs can be calculated using the weakest link principle

$$L_{t,sys,n}^{(farm)} = L_{t,sys,n} \cdot n_{wec}^{-1/c} \tag{3.59}$$

where n_{wec} is the number of WECs in the wave farm.

The above relation is illustrated in Figure 3.5 for $L_{t,sys,n} = 100$ year, and $c = 1.5$. The L10-life for a wave farm, $L_{t,sys,n}^{(farm)}$, is plotted as function of the number of WECs, n_{wec} , which gives a straight line in log-log-scale. For example, for a wave farm with 1000 WECs the L10-values of the time until first failure is 1 year if the L10-life for the ball screw system of each WEC is 100 years.

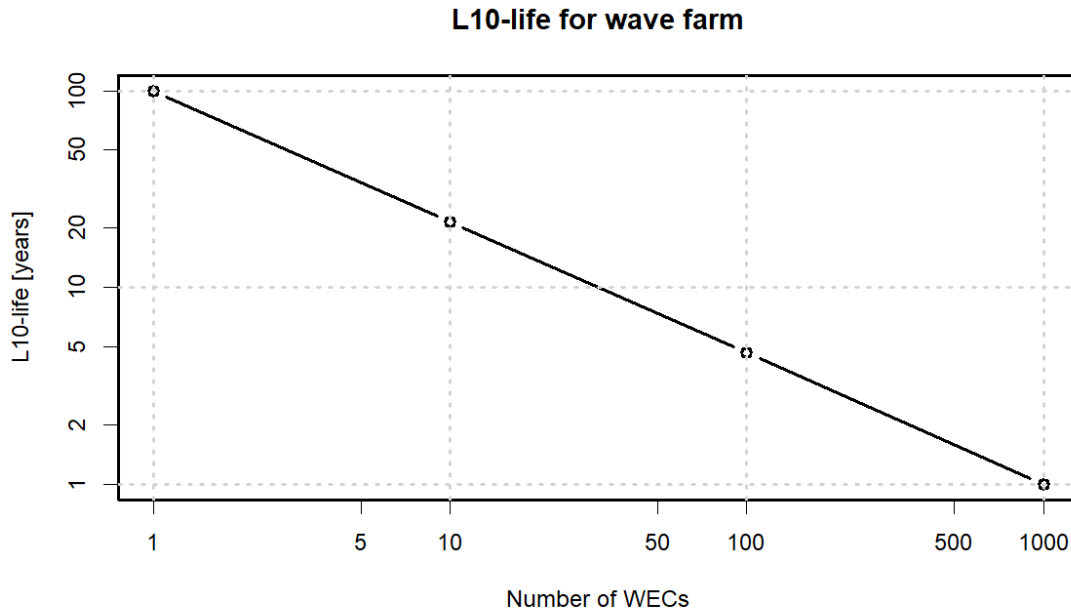


Figure 3.5: Plot of the L10-life of a wave farm, $L_{t,sys,n}^{(farm)}$, as function of the number of WECs in the farm, n_{wec} ; the plot is in log-log-scale.

3.4.4.2 Expected Number of Failures

The life of one WEC with n ball screws sharing the load is modelled by a Weibull distributed random variable denoted by X_n . The characteristic life a_n of X_n can be calculated from the L10-life, using Eqs. (3.56) and (3.46), as follows

$$L_{t,sys,n} = L_{t,1} \cdot n^{3-1/c} \Rightarrow a_n = \frac{L_{t,1} \cdot n^{3-1/c}}{(-\ln 0.9)^{1/c}} \quad (3.60)$$

where $L_{t,1}$ is the L10-life in years for a system with only one ball screw, compare Eq. (3.54). The probability of failure, $P_{f,n,T}$, of the ball screw system of one WEC during usage period T can be calculated as

$$P_{f,n,T} = P(X_n < T) = F_{X_n}(T) = 1 - \exp\left(-\left(\frac{T}{a_n}\right)^c\right) \quad (3.61)$$

see Eq. (3.43). The expected number of failures in a wave farm with n_{wec} number of WECs can now be calculated as

$$n_{f,n,T} = n_{wec} \cdot P_{f,n,T} \quad (3.62)$$

The above relation is illustrated in Figure 3.6 for $L_{t,1} = 10$ year, $c = 1.5$, and $n_{WEC} = 100$. The expected number of failures, $n_{f,n,T}$, in a wave farm of n_{WEC} number of WECs, is plotted as function of the operation period T , for different values of n , the number of ball screws sharing the load in each WEC.

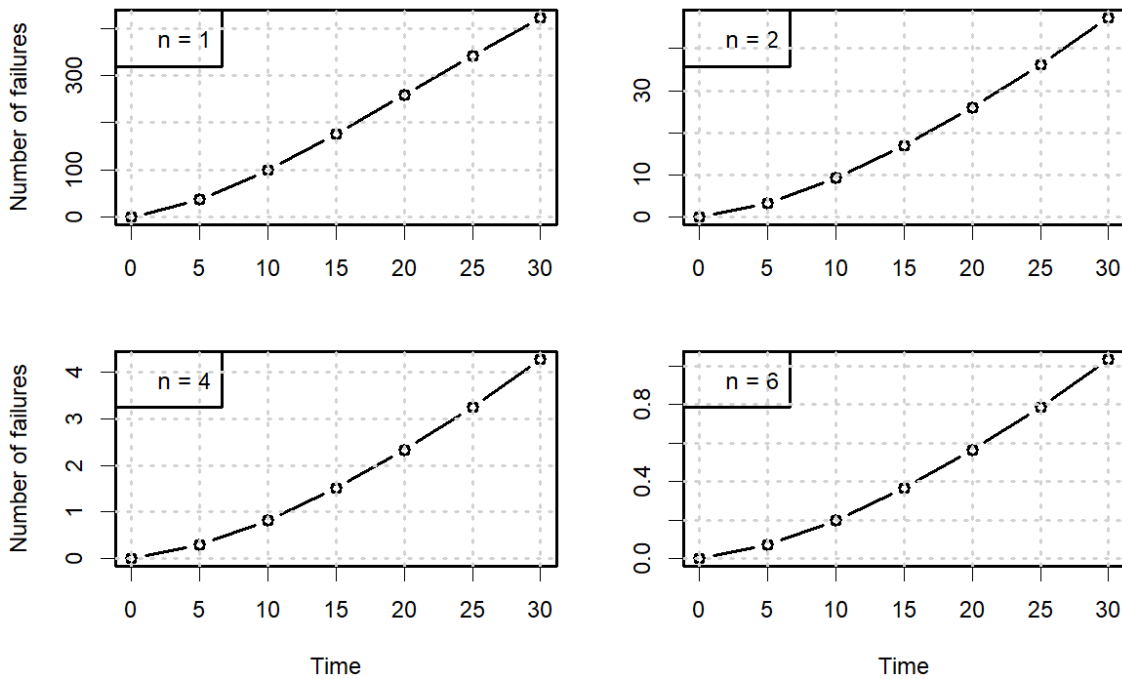


Figure 3.6: The expected number of failures, $n_{f,n,T}$, in a wave farm of 100 WECs, as function of the operation period T , for $n = 1, 2, 4, 6$.

To summarize, the expected number of failures of the ball screw system depends on the number of ball screws in the PTO and the time of operation together with the ball screw specifications and the expected fatigue loads. The evaluation of expected number of failures should serve as an important input to the LCoE calculations for determining suitable service intervals, as well as for choosing the number of ball screws sharing the axial load in each WEC.

4 Assessment of Parameters for Fatigue Model

This section describes the assessment of parameters for predicting the fatigue life of ball screws in the InfinityWEC PTO system. Supplier specifications for estimating fatigue life are presented together with operational limits. Finally, the Weibull distribution is applied to estimate the variability in ball screw life, based on reliability correction factors in ISO standard 3408-5.

4.1 NSK Ball Screw Specifications

The fatigue strength of a ball screw is characterized by its dynamic axial load rating, C_a , and together with other design parameters, its fatigue life can be predicted. The design parameters for the ball screws in the InfinityWEC PTO system are provided by the product supplier, NSK, and are presented in Table 4-1. The dynamic axial load rating is given for ball screws of different sizes, namely, for the full-scale PTO and for the rig-scale (intended for use in the dry-testing campaign in WP4). The rig-scale PTO should represent a 1/4-scale PTO compared to the full-scale PTO. Each scale is available in two types of steel: normal steel or tough steel. Note that the InfinityWEC is under development, therefore the values in Table 4-1 may differ from the final design of the full-scale and rig-scale PTO.

Table 4-1: Model parameters used for the fatigue life model of the ball screws (provided by the product supplier, NSK). Note that the values in the table are preliminary and may differ from the final design of the full-scale and rig-scale PTO.

Parameter	Full-scale		Rig-scale (1/4-scale)	
	Normal steel	Though steel	Normal steel	Though steel
C_a [kN]	1360	1770	22.2	28.9
f_w [-]	1.0			
b [-]	3			
N_0 [cycles]	10^6			
ω_m [rpm]	100			
P_h [m/rev]	0.12		0.08	

The resulting life predictions for variable mean axial force and a constant mean rotational speed of 100 rpm are presented in Figure 4.1, for the four ball screw designs (defined in terms of years or days).

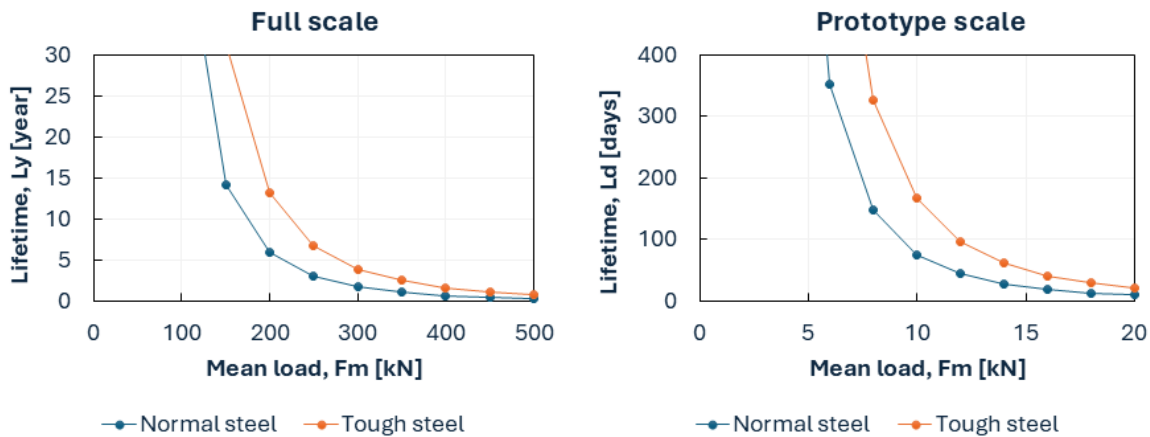


Figure 4.1: Predicted fatigue life of full-scale (left) and rig-scale (right) ball screw designs using two steel types, based on dynamic axial load ratings from NSK. Note that the curves are provided for variable mean load, but with a constant assumed mean rotational speed of 100 rpm.

Note that for a given rotational speed, the lifetime estimate can be converted into the corresponding number of cycles (or revolutions). For example, 100 days in Figure 4.1 corresponds to: $100 \text{ days} \cdot 24 \text{ (h/day)} \cdot 60 \text{ (min/h)} \cdot 100 \text{ (revolutions/min)} = 1.44 \cdot 10^7$ cycles (or revolutions). These curves can be used to estimate the accumulated damage, see Eq. (3.11), based on the number of cycles, calculated as the product of rotational speed and time interval ($\omega_i \cdot \Delta t_i$), and the axial force F_i^b at each increment i . It is noted that these calculations do not account for radial loads or moments.

To ensure reliable operation of the ball screws, certain conditions must be met. These conditions are defined in terms of maximum allowable values for key operational variables. Table 4-2 summarizes a set of critical parameters, including limits for force, temperature, and velocities. Parameters marked as “Ensured in design” refer to those that are guaranteed to remain within safe limits through the design of the PTO system and therefore do not require active monitoring. In contrast, parameters marked as “Monitored” must be continuously tracked during operation to ensure safe and reliable performance.

Table 4-2: Maximum allowable operational parameters for full-scale and rig-scale ball screw designs. Parameters are categorized based on whether they are ensured during the design phase (Ensured in design) or require active monitoring during operation (Monitored) to maintain safe and reliable performance.

Parameter	Maximum allowable value		Remark
	Full-scale	Rig-scale	
Axial speed, $v_{max}^{(axial)}$ [m/s]	2	1	Monitored
Rotational speed, ω_{max} [rpm]	1000	750	Monitored
Axial force, F_{max} [kN]	500 tensile / 325 buckling	16 tensile	Monitored
Radial force, $F_{max}^{(radial)}$ [kN]	5	N/A	Ensured in design
Bending moment, M_{max} [kN*m]	9	N/A	Ensured in design
Temperature, θ_{max} [°C]	70	70	Monitored

According to supplier-provided estimates (Figure 4.2), the moment and radial load acting on the full-scale ball screw during operation must be limited to 9 kNm and 5 kN, respectively. These limits are determined by the mechanical strength of the ball screw and the requirement that an axial force of up to 500 kN must be allowed to act on the joint simultaneously (for the full-scale design). Figure 4.2 presents two graphs showing the estimated lifetime of the full-scale ball screw (using normal steel) under constant axial load and rotational speed, as a function of the applied moment (left) and radial load (right). Within the allowable range, represented by the non-shaded areas, the influence of moment and radial force on fatigue life is minimal and can therefore be considered to be negligible.

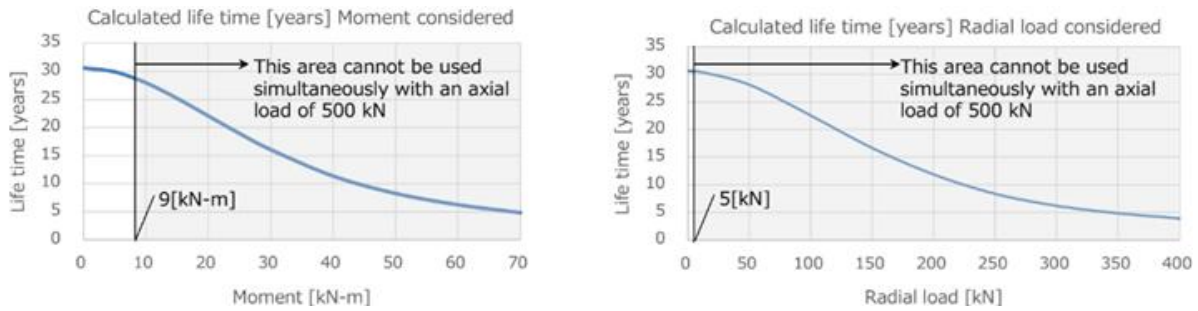


Figure 4.2: Estimated fatigue life of the full-scale ball screw as a function of moment (left) and radial load (right). Non-shaded areas indicate safe operating conditions with minimal impact on lifetime. The results are provided by NSK.

4.2 Weibull Life Distribution

The life of ball screws is often modelled by a Weibull distribution, which is described by two parameters, namely, the characteristic life, a , and the Weibull exponent, c , and has cumulative distribution function (CDF)

$$F_X(x) = 1 - \exp\left(-\left(\frac{x}{a}\right)^c\right) \quad (4.1)$$

see Section 3.4.1 for more details on the Weibull distribution.

Ideally the Weibull parameters are estimated from life testing of ball screws, however this kind of data is scarce. However, ISO 3408-5:2006 provides some general guidance; it gives estimates of the life corresponding to 90% reliability, and indirectly also provides information on the Weibull exponent. First, the Weibull exponent will be derived from information in the ISO standard, and then the median and characteristic life will be derived.

As mentioned in Section 3.3.5, the ISO standard 3408-5:2006 contains a correction factor, f_{ar} , for other reliability levels than 90%, and the corrected life becomes $L_{ar} = L \cdot f_{ar}$. The correction factor can be used to back-calculate the value of the Weibull exponent, c , by solving the equation system

$$\begin{cases} P(X > x_1) = p_1 \\ P(X > x_2) = p_2 \end{cases} \quad (4.2)$$

In the ISO standard the reference is the 90% reliability, giving $f_{ar} = 1$. Now, the two points can be chosen to $(p_1 = 90\%, x_1 = L)$ and $(p_2, x_2 = L \cdot f_{ar})$, where f_{ar} corresponds to reliability level p_2 . By solving Eq. (4.2), the Weibull exponent can then be calculated as

$$c = \frac{\ln\left(\frac{\ln p_2}{\ln 0.90}\right)}{\ln(f_{ar})} \quad (4.3)$$

For reliability level $p_2 = 99\%$, the correction factor is $f_{ar} = 0.21$, and the Weibull shape parameter becomes $c = 1.5$, which should represent a typical Weibull shape parameter for ball screws.

The ISO standard 3408-5 presents the reliability factor for a few reliability levels, however, the correction factor, f_{ar} , for an arbitrary reliability level, p_2 , can be calculated as

$$f_{ar} = \left(\frac{\ln p_2}{\ln 0.9}\right)^{1/c} \quad (4.4)$$

Thus, the characteristic life can be calculated by setting the reliability to $p_2 = e^{-1}$ and using $c = 1.5$, resulting in $a = 4.48 \cdot L$. Further, the median life of the ball screw can be calculated by setting the reliability to $p_2 = 0.5$, giving in $L_{median} = 3.51 \cdot L$.

Note that the relative scatter in life is determined only by the shape parameter, and for $c = 1.5$ the relative standard deviation (or coefficient of variation) becomes

$$\frac{s_W}{m_W} = \sqrt{\frac{\Gamma\left(1 + \frac{2}{c}\right)}{\Gamma^2\left(1 + \frac{1}{c}\right)} - 1} = 0.679 \quad (4.5)$$

which means that the variability of the observed lifetimes is expected to be quite large. Recall Table 3-3, where the mean, the standard deviation, and the relative standard deviation are shown for a Weibull distribution with L10-life, L , and shape parameter, $c = 1.5$.

In order to illustrate the uncertainty in observed ball screw life, a Weibull distribution with L10-life, $L = 1$, and shape parameter, $c = 1.5$, is shown in Figure 4.3. The 95% uncertainty interval is illustrated by the blue area, where the lower and upper limits correspond to the 2.5% and 97.5% quantiles, respectively. The quantiles (2.5%, 50%, and 97.5%) of the distribution are indicated with red lines. Further, uncertainty intervals corresponding to 90%, 95%, 98% and 99% coverage for a Weibull distribution with L10-life, L , and shape parameter, $c = 1.5$, are tabulated in Table 4-3. Recall Figure 3.3 and Table 3-3, where the quantiles 10%, 50%, 90% are shown, corresponding to an 80% uncertainty interval.

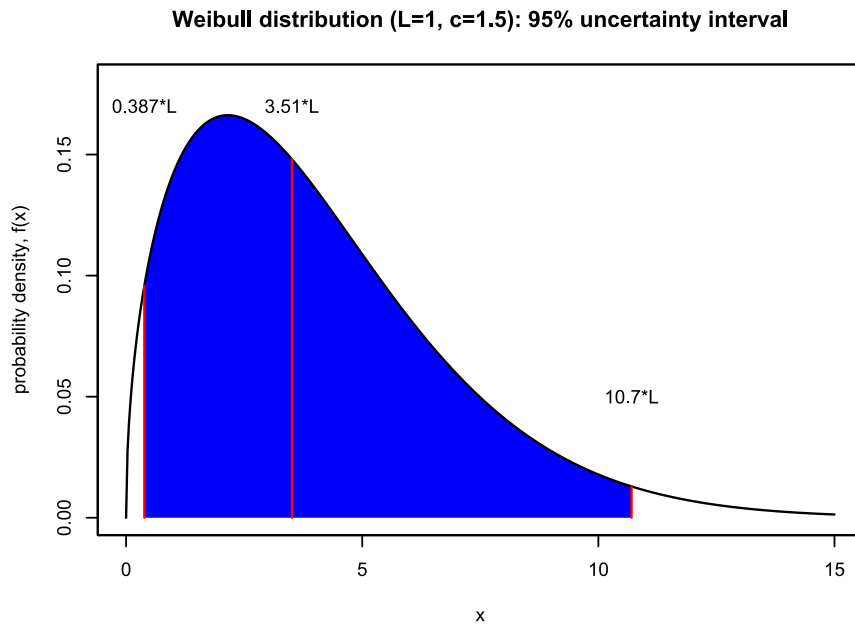


Figure 4.3: PDF of a Weibull distribution with L10-life, $L = 1$, and shape parameter, $c = 1.5$. The 95% uncertainty interval is illustrated by the blue area, and quantiles (2.5%, 50%, and 97.5%) of the distribution are indicated by red lines.

Table 4-3: Uncertainty intervals for a Weibull distribution with L10-life, L , and shape parameter, $c = 1.5$.

Parameter	Quantiles		
	Lower quantile	50%-quantile	Upper quantile
90% uncertainty interval	$0.619 \cdot L$	$3.51 \cdot L$	$9.32 \cdot L$
95% uncertainty interval	$0.387 \cdot L$	$3.51 \cdot L$	$10.7 \cdot L$
98% uncertainty interval	$0.209 \cdot L$	$3.51 \cdot L$	$12.4 \cdot L$
99% uncertainty interval	$0.131 \cdot L$	$3.51 \cdot L$	$13.6 \cdot L$

5 Load and Life Assessment

This section outlines the load assessment required for fatigue life estimation of the ball screw joints in the PTO system. As input to the fatigue life estimation of the ball screws in the PTO system, a detailed load assessment is required to quantify how wave-induced motion translates into mechanical forces. The PTO converts the reciprocating motion of the floating body, typically oscillating in the heave direction, into rotational motion via a ball screw mechanism. This motion subjects the ball screw to cyclic axial loads and torques, which vary in amplitude and direction depending on the wave conditions and control strategy. This setup enables the PTO unit to extract energy from the vertical displacement of waves and convert it into mechanical rotation for power generation.

The process of converting wave-induced motion into mechanical forces acting on the PTO involves formulating the relevant dynamic balance equations and solving for the unknowns. An example with a simplified WEC model is provided in “Appendix A: Calculation Example” to illustrate the process for a sinusoidal wave load, while the detailed analysis for the actual InfinityWEC system is carried out in WP2.

5.1 Design Load Assessment

The design loads for a specific system or component need to be assessed, which is addressed for wave energy applications in e.g. (Atcheson, 2018, 2019). The fatigue loads depend on the site of operation and the operating conditions of the WEC. The wave climate of the site is characterized by the so-called wave scatter diagram, which describes the frequency of occurrence of the different sea states. The response of the WEC to the different sea states can be evaluated through physical testing in e.g. wave tanks or by numerical simulations, e.g. using WEC-Sim. The combination of the responses and the wave scatter diagram together with the target design life gives the estimated total fatigue load on the system or component. The design life can be the full lifetime of the WEC or the replacement interval for wear parts like ball screws.

A sea state is characterized by its significant wave height, H_s , and its wave period, T_p . The wave scatter diagram is a matrix, $F = (f_{ij})$, specifying the frequency of occurrence of each sea state. It can be specified as the number of hours each sea state $(H_{s,i}, T_{p,j})$ occurs during one year (on average), alternatively, it can be normalized to present the relative occurrence in percentage. As an example, the wave scatter diagram representing the Edvard Grieg site in the North Sea is illustrated in Figure 5.1, showing the percentage occurrence of the sea states.

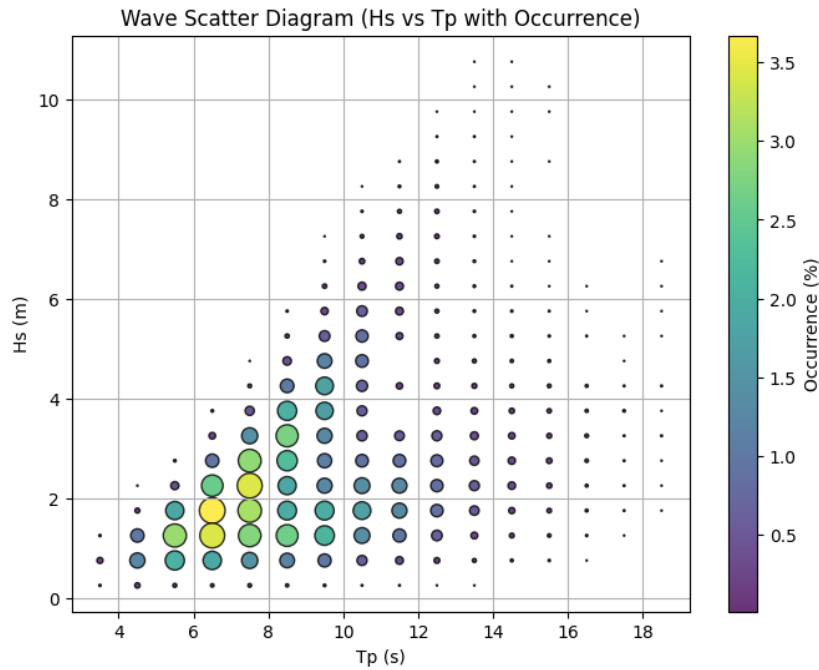


Figure 5.1: Wave scatter diagram representing the Edvard Grieg site.

The procedure for load and life assessment is schematically illustrated in Figure 5.2, and the procedure to estimate the design load will now be described.

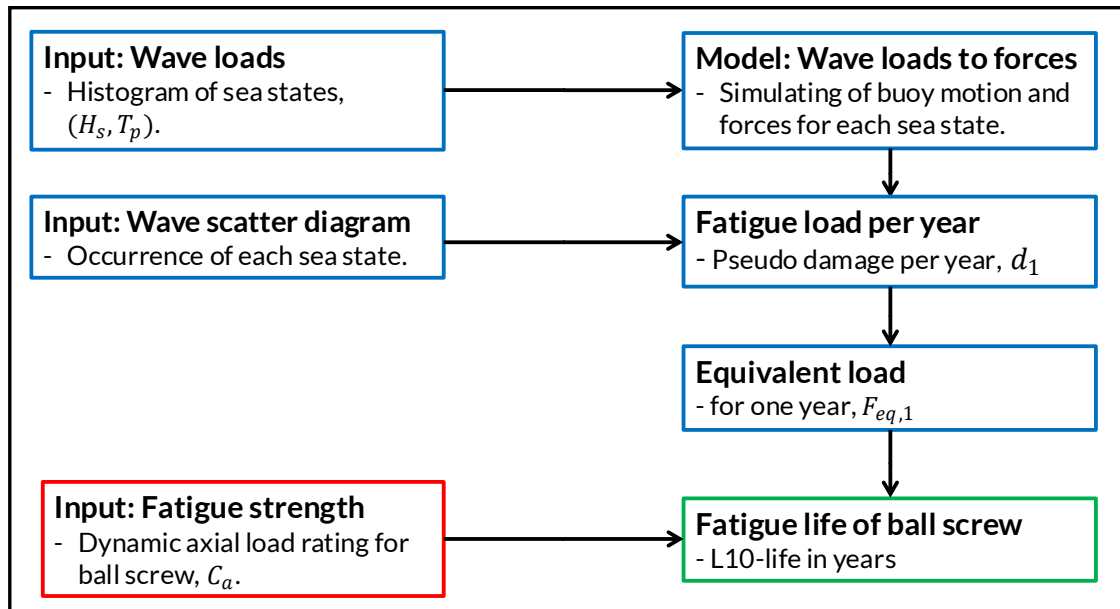


Figure 5.2: A schematical description of the procedure for load and life assessment.

First, the response of the WEC to each sea state $(H_{s,i}, T_{p,j})$ is simulated, resulting in time signals of axial force and rotational speed for the ball screw. The pseudo damage per hour, d_{ij} , for each sea state is then calculated as

$$d_{ij} = \frac{1}{T_{sim}} \sum_{k=1}^n \omega_k \cdot \Delta t_k \cdot F_k^3 \quad (5.1)$$

where T_{sim} is the duration of the simulated signal, see Section 3.3.2. The average pseudo damage for one year can then be calculated by combining the wave scatter diagram and the pseudo damage matrix, denoted $\mathbf{d} = (d_{ij})$, giving

$$d_1 = \sum_{i,j} d_{ij} \cdot f_{ij} \quad (5.2)$$

where d_1 is the one-year pseudo damage, and d_{ij} is the average 1-hour pseudo damage for sea state $(H_{s,i}, T_{p,j})$ in the scatter diagram with annual hours of occurrence, f_{ij} . The one-year equivalent force can now be calculated according using Eq. (3.19) with $T = T_1 = 1$ year

$$F_{eq,1} = \left(\frac{d_1}{N_{eq}} \right)^{1/3} \quad (5.3)$$

where the equivalent number of cycles is $N_{eq} = 10^6$.

Further, the average pseudo damage for the design life, T_{design} , is

$$d_{design} = T_{design} \cdot d_1 \quad (5.4)$$

since damage accumulates linearly over time. The equivalent force for the design life, T_{design} , can be calculated according to Eq. (3.18)

$$F_{eq} = \left(\frac{T_{design} \cdot d_1}{T_1 \cdot N_{eq}} \right)^{1/3} = \left(\frac{T_{design}}{T_1} \right)^{1/3} \cdot F_{eq,1} \quad (5.5)$$

Note that:

- The pseudo damage and the equivalent load depend only on fatigue load, and that they are independent of the load rating, C_a , of the ball screw.
- The pseudo damage increases linearly with time, whereas the equivalent force increases as function of time to the power of 1/3.

For the InfinityWEC, the target design life is aimed for 25 years, thus the fatigue damage for the design life of the WEC is $d_{design} = 25d_1$. However, for wear components, like the ball screws, replacements are scheduled at regular service intervals. For a planned replacement interval of 5 years, the pseudo damage for the design life of the ball screws, representing the replacement interval, is $d_{design} = 5d_1$.

5.2 Life Assessment

Recall the life model presented in Section 3.3, and that the fatigue strength of a ball screw is characterized by its basic dynamic axial load rating, C_a . The L10-life, L , in number of cycles can now be calculated according to ISO 3408-5:2006

$$L = 10^6 \left(\frac{C_a}{F_{eq}} \right)^3 \quad (5.6)$$

where F_{eq} is the equivalent axial force for the design loads corresponding to the design life. The L10-life, L_t , in years is estimated using Eq. (3.21), namely

$$L_t = \frac{T_{design}}{10^6} \cdot L = T_{life} \cdot \left(\frac{C_a}{F_{eq}} \right)^3 \quad (5.7)$$

The L10-life, L_t , can also be calculated from the one-year equivalent force, $F_{eq,1}$,

$$L_t = T_1 \cdot \left(\frac{C_a}{F_{eq,1}} \right)^3 \quad (5.8)$$

with $T_1 = 1$ year.

5.3 An Example of Load and Life Assessment for InfinityWEC G6

This section presents an example of load and life assessment using load response for InfinityWEC G6 (G6A07). Fatigue assessment of a ball screw system with four ball screws is estimated for a mix of representative sea states (received from WP2). It is assumed that the four ball screws are sharing the axial force, acting on the PTO, equally. The loads are obtained by numerical simulation using a linear moment-based controller (denoted MPC). The PTO response for each relevant sea state is simulated, giving 1400 s of useful signals, which was considered sufficient to represent the average power output in the simulations, for each sea state. The results (axial force on PTO, rotational speed, and accumulated and incremental pseudo damage) for the complete 1400 seconds of the load spectrum for the sea state $(H_{s,1.75}, T_{p,6.5})$ are shown in Figure 5.3.

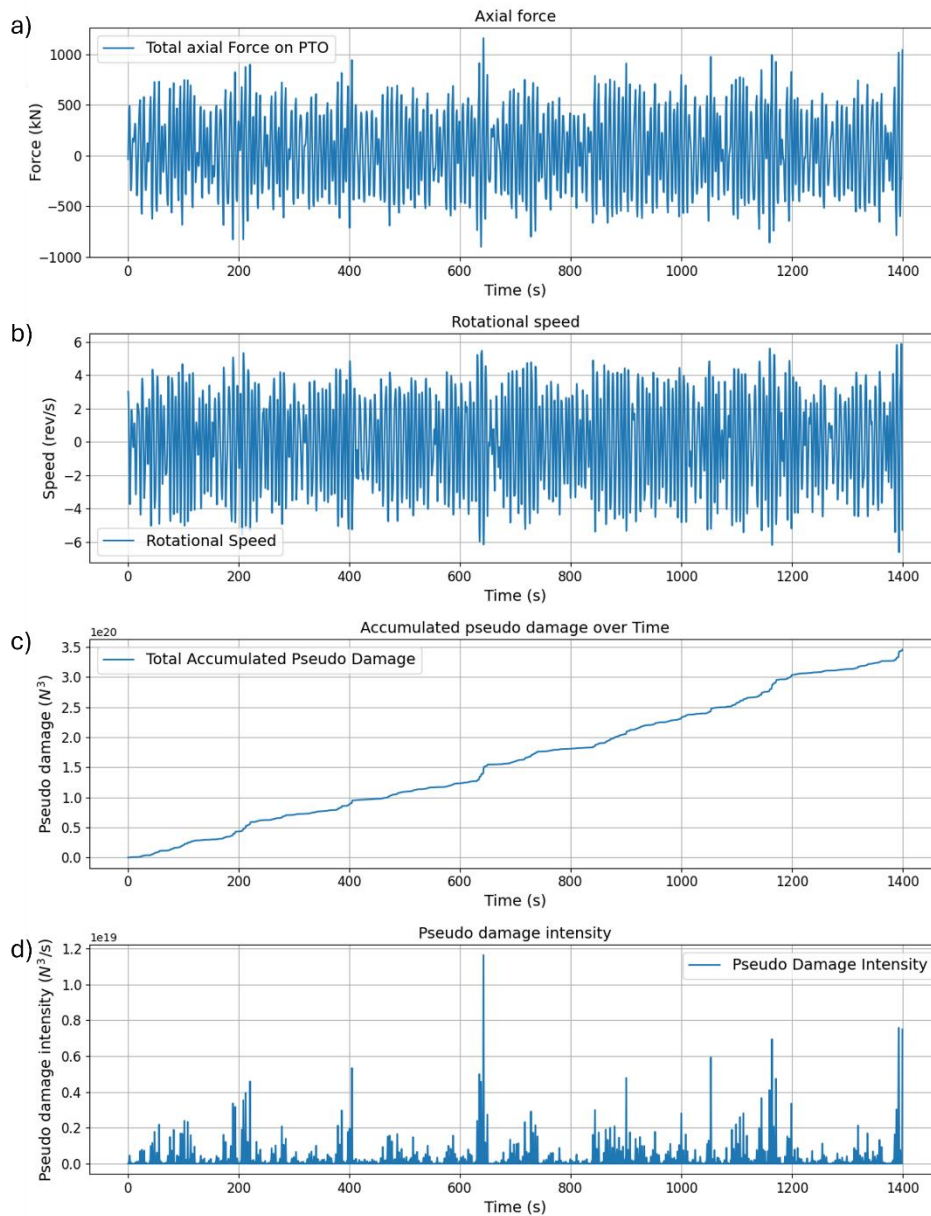


Figure 5.3: Results from simulated 1400 s of the sea state ($H_{s,1.75}, T_{p,6.5}$). a) Total axial force acting on PTO. b) Rotational speed of the ball screws. c) Accumulated pseudo damage, d . d) Pseudo damage intensity, \vec{d}_i .

In Figure 5.4 and Figure 5.5, the accumulated pseudo damage per sea state (d_{ij}) and the accumulated pseudo damage per sea state scaled with the occurrence ($d_{ij} \cdot f_{ij}$) are presented, respectively.

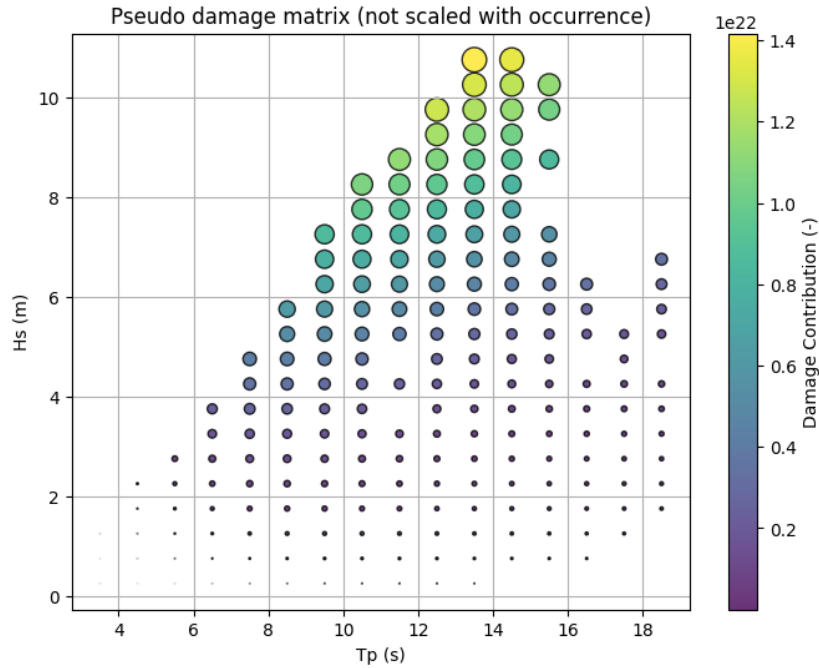


Figure 5.4: Plot showing the pseudo damage per sea state, neglecting occurrence, d_{ij} .

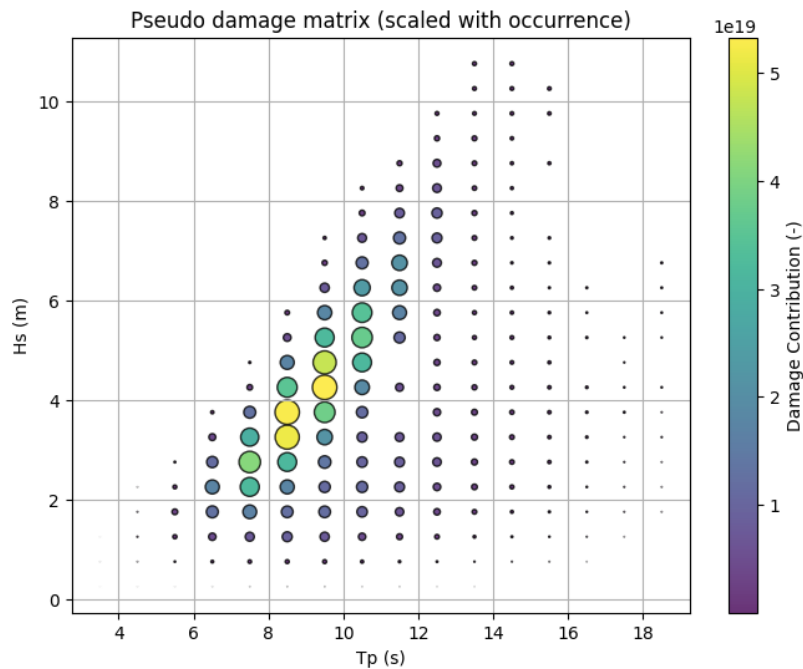


Figure 5.5: Plot showing the pseudo damage, scaled with the occurrence, $d_{ij} \cdot f_{ij}$, per sea state.

To estimate the total accumulated damage over the simulation time interval (1400 s), we sum the scaled damage contributions for all sea states. This value represents the equivalent damage accumulated over 1400 s of operation. Further, to determine the total lifetime (or the required service-interval for the ball screws) of the PTO, we start by estimating the sum of the scaled pseudo damage using Eq. (5.2). For the total axial force acting on the PTO, the corresponding pseudo damage for one year of operation becomes

$$d_1 = \frac{d_{1400s}}{1400} \cdot 3600 \cdot 24 \cdot 365 = 2.7 \cdot 10^{25} \text{ N}^3 \quad (5.9)$$

The one-year equivalent force can now be calculated using Eq. (5.3)

$$F_{eq,1} = \left(\frac{d_1}{N_{eq}} \right)^{1/3} = \left(\frac{2.7 \cdot 10^{25} \text{ N}^3}{10^6} \right)^{1/3} = 3000 \text{ kN} \quad (5.10)$$

Assuming a design life of 5 years, the pseudo damage over the corresponding time can be estimated as $d_{design} = 5d_1$. The equivalent force for the corresponding design life can then be calculated using Eq. (5.5), which for the given case becomes

$$F_{eq} = \left(\frac{T_{design} \cdot d_1}{T_1 \cdot N_{eq}} \right)^{1/3} = \left(\frac{5 \cdot 2.7 \cdot 10^{25} \text{ N}^3}{1 \cdot 10^6} \right)^{1/3} = 5130 \text{ kN} \quad (5.11)$$

with $T_{design} = 5$ year and $T_1 = 1$ year. This value of F_{eq} can now be used to directly compare with the corresponding dynamic axial load rating for the ball screw system, cf. Table 3-4 (in accordance with the conventional ISO standard approach).

In this example, a PTO with $n = 4$ ball screws is assumed. Recall, Eq. (3.57), that the dynamic axial load rating, $C_{a,n}$, for a system of n ball screws with shared load can be calculated as

$$C_{a,n} = C_a \cdot n^{1-\frac{1}{3c}} = C_a \cdot n^{7/9} \quad (5.12)$$

where $c = 1.5$ has been assumed. Using Eq. (5.7), the fatigue life of the ball screw system is obtained, defined in years. For the case of four ball screws in the system, the L10-life is estimated as

$$L_{t,sys,n} = T_1 \cdot \left(\frac{C_{a,n}}{F_{eq,1}} \right)^3 = 1 \text{ year} \cdot \left(\frac{C_a \cdot 4^{7/9}}{3000 \text{ kN}} \right)^3 \quad (5.13)$$

For the full-scale PTO design using normal and tough steel designs with $C_a = 1360 \text{ kN}$ and $C_a = 1770 \text{ kN}$, respectively, (cf. Table 4-1), the L10-life, $L_{t,sys,n}$, is estimated for a system of four ball screws to be 2.3 and 5.1 years, respectively.

Table 5-1 presents load ratings and life estimates for two cases, namely, 1) a PTO with only one ball screw, and 2) a PTO with 4 ball screws sharing the load equally. The estimated life should be compared to the design life of the ball screws, which is the planned maintenance intervals for the InfinityWEC. For the ball screws, it is reasonable to aim for a replacement interval of 5 years. It can be observed that a PTO with only one ball screw does not meet the target, while a PTO with four ball screws of tough steel sharing the load achieves the assumed design target of 5 years.

Table 5-1: Estimated of L10-life for the full-scale PTO design, InfinityWEC G6, for normal and tough steel with design values in Table 4-1, Weibull exponent $c = 1.5$, and one-year equivalent load $F_{eq,1} = 3000 \text{ kN}$.

Ball screw system	One ball screw		System of four ball screws	
Parameter	Load rating	Life	Load rating	Life
Notation	C_a [kN]	$L_{t,sys,1}$ [years]	$C_{a,4}$ [kN]	$L_{t,sys,4}$ [years]
Normal steel	1360	0.091	4000	2.3
Tough steel	1770	0.201	5200	5.1

Note that the estimated life can be affected either by increasing the strength of the ball screw, e.g., by material choice or by design of ball screws, or by decreasing the loads on the ball screw, e.g. by sharing the load on multiple ball screws or by limiting the load using a health-aware control strategy. Further, note that these calculations are based on numerical simulations and ball screw data for InfinityWEC G6, and that the INFINITY project aims at improving the InfinityWEC design as well as its control strategy. The fatigue life of new concepts will be evaluated using the methodology presented in this deliverable, and the results will be presented in coming deliverables of INFINITY.

5.4 Fatigue Load Monitoring

To track and estimate the state of health of the ball screws during operation, the accumulated damage curve should be continuously estimated and monitored, by measuring (or estimating) the axial forces and rotational speeds of the individual ball screws. It is also important to note that the simulation here assumes that the ball screws are equally loaded. Other measured signals, e.g., temperature or vibration, can also be used as indicators for fatigue damage or state of health of the ball screw joints. Structural Health Monitoring (SHM) will be further investigated in INFINITY D5.2.

6 Verification Strategy

This section outlines the strategy for evaluating the developed fatigue life model and exploring structural health monitoring strategies using a rig-scale PTO replica in a dry-testing environment (to be tested in WP4). The approach focuses on validating the model and assessing alternative signals, such as temperature and vibration, alongside force and rotational speed to improve understanding of the degradation processes. Finally, some instructions for accelerated fatigue testing under realistic conditions, to validate life predictions and refine monitoring strategies, is outlined.

6.1 Verification Approach for the Scaled PTO Testing

In WP4, a scaled down version of the PTO system, here called the rig-scale PTO, will be tested in a dry-testing environment using the test rig at VGA. The aim is that the rig-scale PTO should represent a 1/4-scale PTO compared to the full-scale PTO. To evaluate the developed fatigue life model for the ball screws of the rig-scale PTO, an experimental strategy is proposed focusing on tracking the accumulated fatigue damage and exploring alternative measurement techniques for monitoring degradation. An important goal is to determine whether signals beyond axial force and rotational speed, such as temperature and vibration, can serve as useful indicators of degradation in ball screws and PTO components. These additional signals could enable cost-effective and continuous condition monitoring in real operating environments.

The planned test setup consists of a scaled PTO assembly subjected to controlled cyclic axial loading and reciprocating motion, replicating dynamic conditions derived from simulation models (further described in WP4). The accumulated fatigue damage should be estimated and recorded continuously during the test. This measure will serve as reference for comparison with other signals, indicating how the estimated fatigue life of the ball screws evolves in comparison with the complementary responses. It should be noted that values of C_a relevant for the corresponding scale should be used in the assessment. Moreover, since the provided fatigue life from the supplier corresponds to the L10-value (or reliability of 90 percent), the expected experimental life and its uncertainty interval should be calculated using the provided L10-value together with assumed scatter in observed life (e.g., assuming a Weibull distribution with the Weibull shape parameter estimated in Eq. (4.3)). Finally, the fatigue life should be scaled (cf. Section 3.4.2) based on the number of ball screws used in parallel in the scaled-down replica.

Proposed instrumentation to track the accumulated damage as well as complementary signals for assessment are presented as follows:

- Axial force and rotational speed (baseline measurements for fatigue model validation),
- Temperature sensors (to capture thermal effects and frictional heating),
- Vibration sensors (to detect changes in dynamic behaviour linked to wear or surface degradation).
- Generator currents (to evaluate a change in output generated power with respect to a fixed input condition of force and speed).

While accumulated fatigue damage cannot be directly measured during the experiment, condition-based indicators such as force and current variations, temperature trends, and vibration patterns will be analysed to infer degradation progression. The experiment should run for a sufficient number of cycles to allow measurable changes, with duration estimated using the life model developed in this report.

Finally, the verification will compare observed signals and degradation indicators against model predictions, providing insights into:

- The ability to track the accumulated damage according to the fatigue life model.

- The feasibility of using alternative signals for structural health monitoring.
- Potential refinements to model parameters and assumptions for integration into MPC and system-level optimization.

6.2 Accelerated Fatigue Testing

Whether accelerated fatigue testing will be done within the project will be assessed (depending on selected ball screw models and test plan details). This testing is identified as a critical step for future work. Such testing would involve isolating an individual ball screw and subjecting it to high continuous cyclic axial loads until failure, guided by the fatigue model to ensure accelerated damage accumulation. The test should be conducted in an environment that closely replicates real operational conditions, including temperature, possible moisture, and pre-tension force. By applying sufficiently large cyclic forces, the time to failure can be significantly reduced, enabling direct observation of degradation mechanisms and validation of life predictions under realistic conditions.

7 Application of Life Model to INFINITY Project

In this section we briefly present how the fatigue life model will be integrated into the different related parts of the project, such as the control system (in WP2), structural health monitoring (T5.2 & T5.3) and the LCoE assessment (WP7).

7.1 Life Model for Control System

The fatigue life model will be integrated into the control system model (developed in WP2) to enable health-aware optimization of operational settings. By incorporating real-time estimates of fatigue damage progression, the control system can adjust force and motion profiles of the PTO to lower the fatigue damage during the most damaging time periods. This could be achieved by incorporating the damage progression as a penalization in the control algorithm while maintaining performance objectives.

7.2 Life Model for Structural Health Monitoring

The fatigue life model will also serve as a foundation for estimating the remaining useful life (RUL) of the ball screws during operation. To support this, the health monitoring system must continuously collect data on axial force and either rotational speed or axial displacement (the latter converted to rotational speed), of the individual ball screws. These signals will feed into the damage accumulation rule and also help monitor potential violations of operational limits on forces and velocities (cf. Table 4-2). Structural health monitoring is the topic of T5.2, and evaluation of test rig results will be carried out in T5.3.

7.3 Life Model for LCoE Assessment

The LCoE (Levelized Cost of Energy) depends on reliability since more reliable components are generally more expensive but require less maintenance. For the ball screws system of the InfinityWEC, the evaluation of the expected number of failures is presented in Section 3.4.4.2, where it can be seen that the expected number of failures depends on the number of ball screws in the PTO and the time of operation together with the ball screw specifications and the expected fatigue loads. This reliability information should serve as important input to the LCoE evaluations carried out in WP7. It can be used for determining suitable service intervals, but also for choosing an appropriate number of ball screws sharing the axial load in each WEC.

8 Discussion and Conclusions

The fatigue life model for ball screws proposed in this report represents an important step toward improving the reliability and predictive capabilities of the PTO in the InfinityWEC, a novel wave energy converter. By focusing on the ball screw mechanism, the reliability of a critical component subject to complex cyclic loading is addressed. The fatigue life model provides a tailored approach to calculate the fatigue damage accumulation over time. The use of ISO standards ensures methodological robustness, while the possibility of integrating system-specific parameters enhances relevance to the INFINITY project's target application.

This report has developed a fatigue life model for ball screws in PTO systems, tailored to the operational characteristics of wave energy converters. The fatigue life model for ball screws:

- Estimates accumulated fatigue damage over time using rotational speed and axial force signals.
- Adheres to ISO standards while allowing for system-specific parameters to be incorporated.
- Is suitable for integration into the MPC framework to support health-aware control optimization.
- Can be used to define an equivalent fatigue load that can be directly compared to the axial load rating of the ball screw.
- Can be used to assess fatigue loads and fatigue life for design loads.

The main load parameter is the axial force; however, also other load conditions that may impact the life are discussed, e.g., radial force, moment, shock loads, lubrication, and temperature. Further, a statistical model based on the Weibull distribution is used to model the variability in fatigue life. Based on the Weibull distribution and the weakest link principle, the fatigue life is derived for a system of ball screws sharing the axial load, which provides valuable input to the WEC design. Also, the expected number of ball screw failures is evaluated based on the Weibull distribution.

A methodology is presented for assessing the fatigue loads and life for expected service loads, which is exemplified through simulated load signals from the INFINITY G6 WEC. Further, a verification strategy is suggested for evaluating the developed fatigue life model and exploring structural health monitoring strategies using a rig-scale PTO replica in a dry-testing environment (to be tested in WP4).

The findings contribute to the broader goals of the INFINITY project by enhancing the understanding of the fatigue life of ball screws and thus supporting the development of more reliable and cost-effective wave energy systems. Especially, the life model will be used within the further work of INFINITY project, namely:

- *Control system with health-awareness* (WP2): Investigate how the fatigue life model can be incorporated in the control objective.
- *Load and Life Assessment* (WP3): Evaluate fatigue loads and fatigue life for expected service loads.
- *Verification strategy for rig testing* (WP4): Evaluate the fatigue life model and explore structural health monitoring methods.
- *Structural Health Monitoring* (WP5): Develop methods to monitor reliability through tracking accumulated fatigue damage of ball screws.
- *LCoE evaluation* (WP7): Evaluate the impact of control strategy with health-awareness, ball screw system design, and replacement intervals of ball screws.

Assessing how health-aware control influences overall performance and LCoE will be essential for the INFINITY project to ensure both technical and economic benefits.

9 Nomenclature

Table 9-1: Abbreviations.

Abbreviation	Meaning
EC	European Commission
EU	European Union
FMEA	Failure Mode and Effect Analysis
H2020	Horizon 2020
LCoE	Levelized Cost of Energy
MPC	Model Predictive Control
PTO	Power Take-Off
VMEA	Variation Mode and Effect Analysis
WP	Work Package
WEC	Wave Energy Converters

Table 9-2: Variables.

Variable	Description
a	Weibull scale parameter
b	Damage exponent
c	Weibull exponent (shape parameter)
C_a	Basic dynamic axial load rating
$C_{a,n}$	Dynamic axial load rating for a system of n ball screws
d	Pseudo damage
\tilde{d}	Pseudo damage intensity
d_1	One-year pseudo damage
d_{design}	Pseudo damage for design life
d_T	Pseudo damage for duration T
D	Damage
\tilde{D}	Damage intensity
f_{ar}	Reliability factor
f_{ij}	Occurrence frequency of sea state
f_w	Load factor
F	Axial force
$F^{(radial)}$	Radial force

F_0	Fatigue strength (in force)
F_X	Cumulative distribution function
$F_{eq}, F_{eq,1}, F_{eq,T}$	Equivalent force for design life, one year and period T , respectively
F_m	ISO Equivalent axial force
$F_{max}^{(radial)}$	Maximum radial force
F_{max}	Maximum axial force
H_s	Significant wave height
L	Fatigue life in number of cycles
L_t	Fatigue life in years
$L_{sys,n}$	Fatigue life in number of cycles for a system of n ball screws
$L_{t,sys,n}$	Fatigue life in years for a system of n ball screws
m_W	Mean value
M	Moment
M_{max}	Maximum moment
n	Number of (e.g., load cases, cycles, increments or ball screws)
N	Number of cycles (fatigue life)
N_0	Reference number of cycles
N_{eq}	Equivalent number of cycles
P	Probability
P_h	Lead of ball screw
q_j	Subpart in percent
R_X	Reliability function
s_W	Standard deviation
t	Time
T	Duration
T_{design}	Design life
T_p	Wave period
$v^{(axial)}$	Axial speed
$v_{max}^{(axial)}$	Maximum axial speed
X_i	Random variable
θ	Temperature
θ_{max}	Maximum temperature
λ	Scale factor
λ_n	Scale factor for number of cycles

λ_F	Scale factor for force
λ_d	Scale factor pseudo damage (and damage)
$\lambda_{F_{eq}}$	Scale factor for equivalent load
ω	Rotational speed
ω_m	Rotational speed mean value
ω_{max}	Maximum rotational speed

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Appendix A: Calculation Example

This appendix provides a calculation example illustrating the load assessment process for the PTO system. A simplified Cummins equation is used to compute the heave motion of the floating body under sinusoidal wave excitation, and the resulting axial force on the PTO is derived using a (simple) mass-spring-damper model. The example demonstrates how forces and rotational speeds are used to estimate accumulated fatigue damage via the Palmgren–Miner rule, supported by a short simulation and Python implementation.

Hydrodynamic Excitation and Equation of Motion

The motion of the floating body in the heave direction is computed using the simplified form of Cummins' equation:

$$(m + m_{\infty})\ddot{z} + c_{rad}\dot{z} + s_h z = f_{ex} \quad (\text{A.1})$$

Here, z is the heave displacement of the floating body, m is the buoy mass, m_{∞} is the added mass at infinite frequency, c_{rad} is the equivalent radiation damping, and s_h is the hydrostatic restoring coefficient. Moreover, f_{ex} is the wave excitation force (including diffraction and Froude–Krylov components). For more detailed discussion on this topic, we refer to (Ziaei et al., 2024; Ringwood et al., 2014). From this equation we can isolate the acceleration as:

$$\ddot{z} = \frac{f_{ex} - c_{rad}\dot{z} - s_h z}{(m + m_{\infty})} \quad (\text{A.2})$$

We can now use the Euler method to solve the ordinary differential equation (ODE) i.e., by approximating the solution over discrete time steps. The updated velocity and displacement over the duration Δt can therefore be approximated as:

$$\dot{z}(t + \Delta t) = \dot{z}(t) + \ddot{z}(t)\Delta t, \quad z(t + \Delta t) = z(t) + \dot{z}(t)\Delta t \quad (\text{A.3})$$

It should be noted that the acceleration and velocity are assumed to be constant over the (small) time step Δt . The mechanical response of the PTO can further be modelled as a mass-spring-damper system (assuming that the translator in the PTO follows the heave motion z). A simplistic model for the axial force acting on the PTO, and thus on the ball screw, can be expressed as:

$$f_{PTO} = m_{eff}\ddot{z} + c_{mech}\dot{z} + k_{mech}z \quad (\text{A.4})$$

where m_{eff} is the effective moving mass of the translator², c_{mech} is the mechanical damping coefficient, k_{mech} is the stiffness of the PTO linkage. This force represents the internal mechanical load resulting from the wave-induced motion and can be solved by introducing mechanical parameters of the PTO as well as the heave displacement z , velocity \dot{z} and

² It is noted that $m + m_{\infty}$ corresponds to the total inertial resistance of the floating body to acceleration (hydrodynamic domain), while m_{eff} refers to the mass of the internal moving components (such as pistons, sliders, or other components that move linearly) that follow the floating body's motion and transmit force to the ball screw.

acceleration \dot{z} . We note that although torque can be derived from the axial force and screw geometry, it is not required for the fatigue life model used here, which relies solely on axial force and rotational speed as inputs.

Load Spectrum and Fatigue Assessment

Given the formulation of the damage rule, the time history of $f_{PTO}(t)$ and $\dot{z}(t)$ can be directly used to compute accumulated fatigue damage using the Palmgren–Miner rule in Eq (3.9).³ In terms of data for the life model, this corresponds to a varying axial force F_i^b and rotational speed (ω_i), for each increment i , as function of the wave loads (sea state). In this PTO system, four parallel-connected ball screws convert vertical wave-induced motion (heave) into rotational motion. Hence, the axial force and rotational speed per screw for the i :th increment may be estimated as

$$F_i^b = \frac{f_{PTO,i}}{4}, \quad \omega_i = \frac{\dot{z}_i}{P_h} \quad (\text{A.5})$$

where: P_h is the lead of the screw (i.e., the linear distance moved per revolution). By summing up the contributions, the accumulated damage metric (D) can be used in the fatigue life model to estimate the remaining useful life (RUL) of the ball screw joints.

Simple Sinusoidal Wave Force

To demonstrate the load assessment process a simple example is provided. As load, we use a sinusoidal wave force to represent the wave-induced excitation acting on the floating body. In Figure A.0.1, results from an example simulation is shown, for which a sinusoidal wave excitation with a force amplitude of 1000 N and frequency of 0.2 Hz is applied (and with model parameters listed in Table A-1).

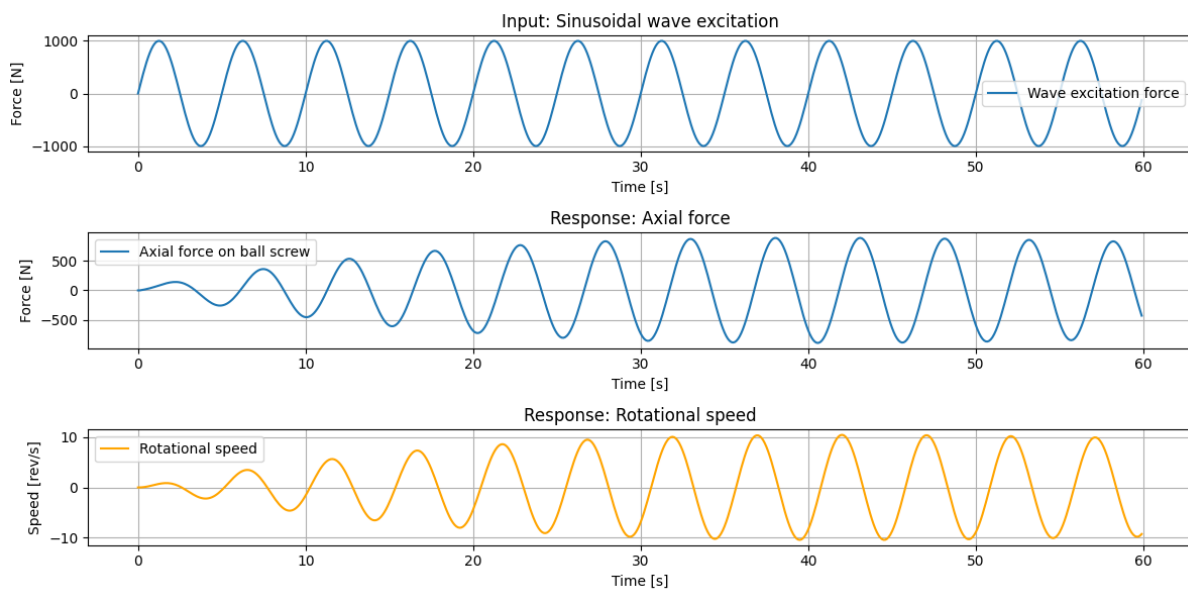


Figure A.0.1. Results for an example simulation, for which a sinusoidal wave excitation with a force amplitude of 1000 N and frequency of 0.2 Hz is applied.

³ Note: Damage accumulation is defined in terms of revolutions (represented as cycles). Hence, no conversion of cyclic loading is required (e.g. via Rainflow counting).

The wave force amplitude (input) is shown together with the response variables: axial force (F) acting on the ball screws and rotational speed (ω) of the ball screws. It should be noted that the estimated rotational speed (based on the estimated velocity) is noticeably affected by the assumed lead of the screw (P_h). In this example it is assumed to 0.12 m. Similar procedure can be used for more complex loading scenarios.

Table A-1. Model parameters for estimating the loads (used for the simulation example).

Parameter	Value	Description	Unit
Δt	0.1	Time step	s
t_{sim}	60	Total simulation time	s
W_{freq}	0.2	Wave frequency	Hz
f_{amp}	1000	Wave force amplitude	N
m	5000	Buoy mass	kg
m_{∞}	2000	Added mass	kg
s_h	10000	Hydrostatic restoring coefficient	N/m
c_{rad}	500	Radiation damping	Ns/m
m_{eff}	1000	Effective moving mass	kg
c_{mech}	300	Mechanical damping	Ns/m
k_{mech}	5000	Mechanical stiffness	N/m

Damage Accumulation

Given the load profiles for the axial force (F) acting on the ball screws and rotational speed (ω) or the ball screws, we can now estimate the accumulated damage by summing up the contributions as:

$$D = \frac{1}{N_0 \cdot C_a^b} \cdot \sum_{i=1}^n \omega_i \cdot \Delta t \cdot F_i^b \quad (\text{A.6})$$

It is worth noting that the angular velocity from the signal shown in Figure A.0.1 is defined in rev/s. The accumulated damage for this short-wave load sequence for the full-scale set up, using normal steel (cf. Table 4-1), is estimated to be $1.84 \cdot 10^{-14}$. In this example we used a time step of $\Delta t = 0.1$ s and a total duration of 60 s. Note that the used parameters are not corresponding to the actual PTO design in the INFINITY project.

Degradation Classification

This approach aligns with the model-free factor-dependent degradation (MF+FD) framework described in (Ziaei et al., 2024). In this classification, degradation is modelled as a function of system variables such as the control input $u(t)$, the heave displacement $z(t)$, and the velocity $\dot{z}(t)$. Here, $u(t)$ specifically denotes the control force applied by the PTO system, which actively regulates the motion of the wave energy converter. Although these variables influence the mechanical stress and fatigue accumulation in components like the ball screw, their effects on system behaviour are not directly observable until degradation reaches a critical threshold. This makes the MF+FD framework particularly suitable for fatigue modelling based on

accumulated damage and enables integration of health-sensitive control strategies into the MPC framework.

Python Code

The Python code used for this calculation example is provided below.

```
"""
Ball screw fatigue simulation under sinusoidal wave excitation
-----
This script provides a simplified estimate of the dynamic response of a heaving
buoy system connected to a ball screw-based Power Take-Off (PTO) mechanism. It
models the motion induced by sinusoidal wave excitation and calculates the
resulting axial force and rotational speed of the ball screw. Finally, the
simulation estimates the fatigue damage using the accumulated damage approach.
Key Components:
-----
- Wave excitation modelled as a sinusoidal force input.
- Heave motion response based on Cummins equation (simplified).
- Axial force on the PTO translator derived from mechanical parameters.
- Rotational speed of the ball screw calculated from axial velocity and screw lead.
- Fatigue damage estimated using Palmgren-Miner rule.
Outputs:
-----
- Time series plots of wave excitation force, axial force, and rotational speed.
- Estimated accumulated fatigue damage.
- Maximum values for rotational speed (rpm), axial displacement (m/s), and axial
force (kN).
Dependencies:
-----
- numpy
- matplotlib
Date: [2025-09-18]
"""

import numpy as np
import matplotlib.pyplot as plt

# Simulation parameters
dt = 0.1 # time step [s]
t_max = 60 # total simulation time [s]
t = np.arange(0, t_max, dt)

# Wave excitation force (simplified sinusoidal input)
wave_freq = 0.2 # Hz
F_exc = 1000 * np.sin(2 * np.pi * wave_freq * t) # [N]

# Cummins-based heave motion response (simplified)
M = 5000 # buoy mass [kg]
A_inf = 2000 # added mass [kg]
h = 10000 # hydrostatic restoring coefficient [N/m]
c_rad = 500 # radiation damping [Ns/m]

# Initialize motion arrays
z = np.zeros_like(t) # displacement [m]
z_dot = np.zeros_like(t) # velocity [m/s]
z_ddot = np.zeros_like(t) # acceleration [m/s^2]

# Time integration (Euler method)
for i in range(1, len(t)):
    z_ddot[i] = (F_exc[i-1] - c_rad * z_dot[i-1] - h * z[i-1]) / (M + A_inf)
```

```
z_dot[i] = z_dot[i-1] + z_ddot[i] * dt
z[i] = z[i-1] + z_dot[i] * dt

# Axial force on PTO translator
M_eff = 1000 # effective moving mass [kg]
c_mech = 300 # mechanical damping [Ns/m]
k_mech = 5000 # mechanical stiffness [N/m]
F_axial = 0.25*(M_eff * z_ddot + c_mech * z_dot + k_mech * z)

# Rotational speed of ball screw
lead = 0.12 # screw lead [m/rev]
omega = z_dot / lead # [rad/s]

# Plot results
plt.figure(figsize=(12, 6))
plt.subplot(3, 1, 1)
plt.plot(t, F_exc, label='Wave excitation force')
plt.xlabel('Time [s]')
plt.ylabel('Force [N]')
plt.title('Input: Sinusoidal wave excitation')
plt.grid(True)
plt.legend()

plt.subplot(3, 1, 2)
plt.plot(t, F_axial, label='Axial force on ball screw')
plt.xlabel('Time [s]')
plt.ylabel('Force [N]')
plt.title('Response: Axial force')
plt.grid(True)
plt.legend()

plt.subplot(3, 1, 3)
plt.plot(t, omega, label='Rotational speed', color='orange')
plt.xlabel('Time [s]')
plt.ylabel('Speed [rev/s]')
plt.title('Response: Rotational speed')
plt.grid(True)
plt.legend()

plt.tight_layout()
plt.show()

# S-N curve parameters (full-scale, normal steel)
Ca = 1360000 # [N]
N0 = 1e6 # cycles to failure
w_mean = 100 # Mean rotational speed used for the SN-curve

# Calculate accumulated damage
def calculate_damage(F_axial, omega, dt, N0, Ca):
    damage = 0
    for F, w in zip(F_axial, omega):
        # print(abs(w)*dt)
        damage += abs(w) * dt * (abs(F) ** 3)
        # print(damage)
    damage /= (N0 * Ca**3)
    return damage

# Run the procedure
damage = calculate_damage(F_axial, omega, dt, N0, Ca)

print(f"Estimated accumulated fatigue damage (Palmgren-Miner): {damage:.2e}")
```